

FUSION OF SAR AND OPTICAL IMAGES FOR REMOTE SENSING APPLICATIONS

*A Project report submitted in partial fulfilment of the requirements for
the award of the degree of*

**BACHELOR OF ENGINEERING
IN
ELECTRONICS AND COMMUNICATION ENGINEERING**

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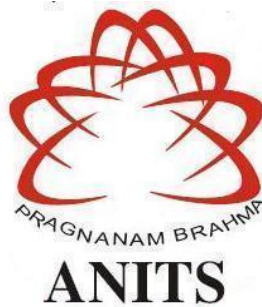
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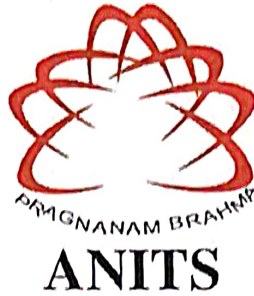
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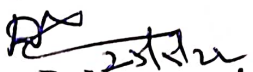
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CERTIFICATE

This is to certify that the project report entitled "FUSION OF SAR AND OPTICAL IMAGE FOR REMOTE SENSING APPLICATIONS" submitted by R.N.S.NIKHIL (318126512044), K.R.L.PRAVALLIKA (319126512L01), H.MAHIMA (318126512020), G.BHARGAVI (319126512L06) in partial fulfilment of the requirements for the award of the degree of **Bachelor of Technology in Electronics & Communication Engineering** of Anil Neerukonda Institute of Technology and Sciences, Visakhapatnam is a record of bonafide work carried out under my guidance and supervision.


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ABSTRACT

Image fusion is a technique in which two or more images are combined to generate a single image that has the important properties of both images. In this project the images are taken from SAR and optical. The SAR is abbreviated as "Synthetic Aperture Radar" and it is a well-established remote sensing technology used worldwide for several applications. It has the possibility of sensing earth at day and night in any weather conditions. An optical satellite has a great view on a large portion of area, however optical sensors measure reflected solar light, thus only functioning in the daytime. The project is based on considering images from these sources and by using Feature level fusion techniques to get the resultant images having both spectral and spatial information. In Feature level fusion, prominent features like edges, corner points, contours, texture, and shapes are extracted from different images and a combined feature map is produced, different transforms like discrete wavelet, Gram Schmidt transform can be used for feature level extraction and their performance with respect to fusion is compared in this project. Wavelet transforms offer a simultaneous localization in time and frequency domain and the other advantage is it is computationally very fast and able to separate the fine details in an image. In this project the fusion using the Gram-Schmidt transform and wavelet transform is compared with the fusion of SAR and optical images using the wavelet transform by finding out the parameters such as information entropy, cross entropy, average gradient, correlation coefficient.

CONTENTS

ACKNOWLEDGEMENT	i
ABSTRACT	ii
LIST OF FIGURES	v
LIST OF ABBREVIATIONS	vii
LIST OF TABLES	vii
CHAPTER-1 INTRODUCTION	1
1.1 Overview	2
1.2 Literature survey	3
1.3 Organisation of project report	5
CHAPTER-2 IMAGE PROCESSING	6
2.1 Introduction	7
2.2 Digital image processing	8
2.3 Advantages	13
2.4 Applications	13
CHAPTER-3 IMAGE FUSION	14
3.1 Introduction	15
3.2 Need for image fusion	19
3.3 Types of image fusion	19
3.4 Image fusion techniques	22
CHAPTER-4 WAVELET TRANSFORM AND GRAM-SCHMIDT	27
4.1 Introduction	28
4.2 Different types of wavelets	30
4.3 Wavelet Transforms	33
4.4 Types of Wavelets Transform	36
4.4.1 Discrete Wavelet transform	37

4.4.2 Dual Tree Complex Wavelet Transform	39
4.4.3 Continuous Wavelet Transform	41
4.4.4 Stationary Wavelet Transform	43
4.4.5 Biorthogonal Wavelet transform	44
4.5 Gram-Schmidt transform	46
4.5.1 Projections	46
CHAPTER-5 SIMULATION TOOL	49
5.1 Introduction	50
5.2 Math-works	50
5.3 MATLAB functions	51
5.3.1 Read, write and show image	51
5.3.2 Image reverse	52
5.3.3 Gram-Schmidt functions	53
5.4 MATLAB toolbox	54
5.4.1 Wavelet Toolbox functions	55
CHAPTER -6 PROJECT DESCRIPTION	56
6.1 Wavelet Decomposition	57
6.2 HAAR Wavelet Transform	58
6.3 Gram-Schmidt Transform	58
6.4 Fusion Technique	60
6.5 Fusion flowchart	60
6.6 Algorithm	61
6.7 Performance Analysis	61
CHAPTER -7 RESULTS	63
7.1 Parameter analysis and output fused images	64
CONCLUSION	67
REFERENCES	68

LIST OF FIGURES

Fig 2.1 Example of Image Processing	07
Fig 2.2 Effect of digitization	09
Fig 2.3 Scaling	10
Fig 2.4 Magnification	10
Fig 2.5 Rotation	11
Fig 2.6 Contrast Stretching	12
Fig 2.7 Noise Filtering	12
Fig 3.1 Example for image fusion	15
Fig 3.2 Pixel level fusion	17
Fig 3.3 Feature level fusion	18
Fig 3.4 Decision level fusion	18
Fig 3.5 Multimodal fusion	20
Fig 3.6 Multitemporal fusion	21
Fig 3.7 Multifocus fusion	21
Fig 3.8 Fusion for Image Restoration	22
Fig 4.1 Example of a wavelet	28
Fig 4.2 Haar wavelet function	31
Fig 4.3 Daubechies wavelet	32
Fig 4.4 Mexican Hat wavelet	33
Fig 4.5 Representation of signal in different scales and positions	33
Fig4.6 Illustration of short time fourier transform	35

Fig 4.7 Time resolution of STFT and WT	36
Fig. 4.8 Discrete wavelet transform	37
Fig 4.9 Block diagram of DWT	38
Fig.4.10 Time and frequency plane of the wavelet transform.	39
Fig 4.11 Block diagram of DT-CWT	40
Fig.4.12 Continuous wavelet transform of a signal	42
Fig 4.13 Illustration of Stationary wavelet transform	43
Fig 4.14 Block diagram of SWT	44
Fig 4.15 Biorthogonal Wavelets	45
Fig 4.16 Procedural step of Biorthogonal Wavelet Transform	45
Fig 4.17 Gram-Schmidt Transform projection	46
Fig 5.1 Image Reversal	52
Fig 5.2 Shifted Image	53
Fig 6.1 Wavelet decomposition of the image	57
Fig 6.2 Haar wavelet transform	58
Fig 6.3 Gram-Schmidt transform	59
Fig 6.4 Image Fusion method combining Gram-Schmidt and Haar Wavelet Transform	60
Fig 7.1 Original and fused images of set 1	64
Fig 7.2 Original and fused images of set 2	65
Fig 7.3 Original and fused images of set 3	66

LIST OF ABBREVIATIONS

DWT: Discrete Wavelet Transform
GS: Gram-Schmidt Transform
PCA: Principal Component Analysis
FIR: Finite Impulse response
SAR: Synthetic Aperture Radar
NSCT: Non-Subsampled Contourlet Transform
FSD: Filter Subtract Deciform
SWT: Stationary Wavelet Transform
CWT: Continuous Wavelet Transform
STFT: Short Time Fourier transform

LIST OF TABLES

Table 7.1	Parameter Analysis of set 1 images	64
Table 7.2	Parameter Analysis of set 2 images	65
Table 7.3	Parameter Analysis of set 3 images	66

CHAPTER -1

INTRODUCTION

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

Image fusion is one of the most widely used image processing techniques. We can produce high-resolution multispectral images by various image fusion methods. There have been a variety of approaches in image fusion such as image overlay, image sharpening, and image cueing through pixel, feature or region/shape combinations. By applying these techniques, the images have different contextual information, and generalised metrics of image fusion gain. In this paper, we intend to find out the various image fusion techniques.

Several situations in image fusion are required for medical, industrial, and remote sensing applications. The process of image fusion is to combine information from multiple images of the same scene into a single image that ideally contains all the important features from each of the original images. The resulting fused image will be thus more suitable for human and machine perception or for further image processing tasks. Image acquisition can be established by different sources like camera, scanner, and other imaging sensors, thus the output information about the scene is achieved. This scene comprises e.g., objects, events, or more complex situations. The output quantities in an output image can be images, features, or symbolic information such as decisions. In automated visual inspection, the ultimate aims in most cases are to gain macroscopic geometrical information (e.g., length, width, or position of an object), to characterise the surface (e.g., reflectance properties, roughness, microstructure, occurrence of surface defects like dents, grooves or other marks), or to obtain volume properties of an object (e.g., material classification, degree of transparency, spatial distribution of components or defects). This information can then be used in various ways in industrial production engineering, e.g., for quality inspection or materials management.

However, many relevant scene properties cannot be determined automatically by evaluating just one image. Instead, the information of interest can often be captured in an image series by means of a properly designed imaging setup using homogeneous or inhomogeneous imaging sensors. The task of image fusion is then to collect and combine the desired information from the image series by means of an adequate extraction of the useful information. This approach has a direct correspondence to the common visual examination performed by a human in everyday life if a human is not able to determine the property of interest at first glance. Besides this essential justification for the use of image fusion in many situations, there are also several other task-dependent reasons.

1.2 LITERATURE SURVEY

SAR and Optical are the two main data types of remote sensing platforms. Optical remote sensing data have the ability to reflect real texture, colour and geographical distribution information, hence the image can be easily analysed, but the quality of image can be limited by weather and time. This type of sensor generates the image over a large area. SAR is a form of radar, which dispatches a microwave pulse off to the earth to determine the physical properties, it does not contain spectral information and easily produce layover shadow and other phenomenon that affects the observation. Fusion of these two images will provide a high-resolution image.

1. Wavelet based fusion of SAR and Panchromatic images for urban areas

In this paper, a fusion method is proposed, which provides an integrated map of an urban area starting from co-registered optical (panchromatic and multispectral) and SAR images. Therefore, a wavelet transform method is used to fuse the SAR and Panchromatic image.

2. Pixel level fusion techniques for SAR and Optical images

This paper discusses the necessity of fusing synthetic aperture radar (SAR) and optical imagery. A survey is presented for various pixel level approaches used for the fusion of SAR and optical images. Quality metrics used to evaluate the performance of a fusion algorithm, are briefly introduced and visual as well as quantitative evaluation of basic component substitution and wavelet-based fusion approaches is presented for the fusion of RISAT-1 SAR and Resourcesat-2 multispectral data. Finally, the review concludes that there is scope for further research of fusion of SAR and Optical images due to various microwave and optical sensors with the improved resolution being launched regularly.

3. Registration of SAR and Optical images by weighted SIFT based on phase congruency

In this paper, to address problems in the registration of synthetic aperture radar (SAR) and optical images due to large grey differences, the scale-invariant feature transform (SIFT) approach based on phase congruency (PC-SIFT) is proposed. This approach is used to address the gradient inversion in multi-source images, and it is based on optimising the dominant direction interval of the descriptors. We construct a new descriptor by combining phase consistency and the gradient amplitude, which is referred to as PCG-SIFT descriptor. The proposed algorithm is suitable for multi-sensor images with large gray differences and significant edge features. The results of experiments show that compared to the traditional gradient-based SIFT descriptor, the PC-SIFT descriptor improve the robustness and matching probability of the registration algorithm for multi-source images

4. Fusion of GF-3 SAR and Optical images based on the Nonsubsampled Contourlet Transform

The fusion of GF-3 SAR images with the multispectral images can improve the visual quality of the SAR images. We show how to use the nonsubsampled contourlet transform(NSCT) for simulating high-resolution images such that both the details of

the SAR image and the spectral information of the multispectral image can be retained. To verify the effectiveness of the proposed idea, two types of resolutions are used and the experimental data: the GF-3 satellite SAR images with resolutions of 3m and 5m, respectively and the GF-1 satellite multi-spectral images with a resolution of 16m ,then perform comparative experiments with different fusion algorithms. The results show the effectiveness of the proposed approach. The traditional method that directly fuses the SAR and multi-spectral images remains. The NSCT average images and the average NSCT images can retain the spectral information. The spectral information of NSCT average images is closer to the multi-spectral images than the average NSCT images.

5. A Fusion of SAR Image And Optical Image Based on NSCT And Gram-Schmidt Transform

In this paper, the GS transform method is improved with non-subsampled contourlet transform (NSCT) which is used to obtain a high-resolution image that contains both spectral information and SAR image detail information. The results show that the spectral information of the fusion image is well preserved, and the indicators are better than other methods.

1.3 ORGANISATION OF THE PROJECT REPORT

The project report is organised in eight chapters. It starts with the introduction. The current chapter introduces the project and gives a brief description of image fusion. Chapter 2 provides information about image processing, its applications, and advantages. Chapter 3 gives a detailed explanation about Image fusion, principles, and methods. Chapter 4 explains about the Wavelet and its types and gives information about the Gram-Schmidt transform. Chapter 5 explains about the software tool MATLAB. Chapter 6 presents the project description, parameters of image fusion, algorithm and flowchart of the project and the results and the conclusion.

CHAPTER-2

IMAGE PROCESSING

CHAPTER-2

IMAGE PROCESSING

2.1 INTRODUCTION:

Image processing is a method to perform some operations on an image, in order to get an enhanced image or to extract some useful information from it. It is a type of signal processing in which input is an image and output may be an image or characteristics/features associated with that image. Nowadays, image processing is among rapidly growing technologies. It forms a core research area within engineering and computer science disciplines too.

Image processing basically includes the following three steps:

- 1.Importing the image via image acquisition tools
- 2.Analysing and manipulating the image
- 3.Output in which the result can be an altered image or report that is based on image analysis.



Fig 2.1 Example of Image Processing

Types :

There are two types of methods used for image processing namely:

- **Analogue image processing:**

It can be used for hard copies like printouts and photographs. Image analysts use various fundamentals of interpretation while using these visual techniques.

- **Digital image processing:**

This technique helps in manipulation of the digital images by using computers. The three general phases that all types of data have to undergo while using digital techniques are pre-processing, enhancement, and display, information extraction.

2.2 DIGITAL IMAGE PROCESSING TASKS:

Image Reconstruction

- Removal of system or imaging aberrations.
- Aims to reconstruct the best image from collected data.
- Typically output images for visual inspection.

Image Analysis

- Computer analysis of images.
- Extract features or regions
- Recognition of objects.
- High level pattern recognition.

Image Formation

Image formed by computer: the image generated by the computer for various purposes.

Image Compression and Encoding

Document and image storage: Compressing and storing a formed image.

Image Representation :

An image defined in the “real world” is considered to be a function of two real variables, for example $f(x, y)$ with f as the amplitude (e.g., brightness) of the image at the real coordinate position (x, y) .

The effect of digitization is shown in Fig 2.2

The 2D continuous image $f(x, y)$ is divided into N rows and M columns. The intersection of a row and a column is called a pixel. The value is assigned to the integer coordinates $[m, n]$ with $\{ m=0,1,2, \dots, M-1 \}$ and $\{ n=0,1,2, \dots, N-1 \}$ is $f[m, n]$. In fact , in most cases $f(x, y)$ -- which we might consider to be the physical signal that impinges on the face of a sensor. Typically, an image file such as BMP, JPEG, TIFF etc., has some header and picture information. A header usually includes details like format identifier (typically first information), resolution , number of bits/pixels , compression type, etc.

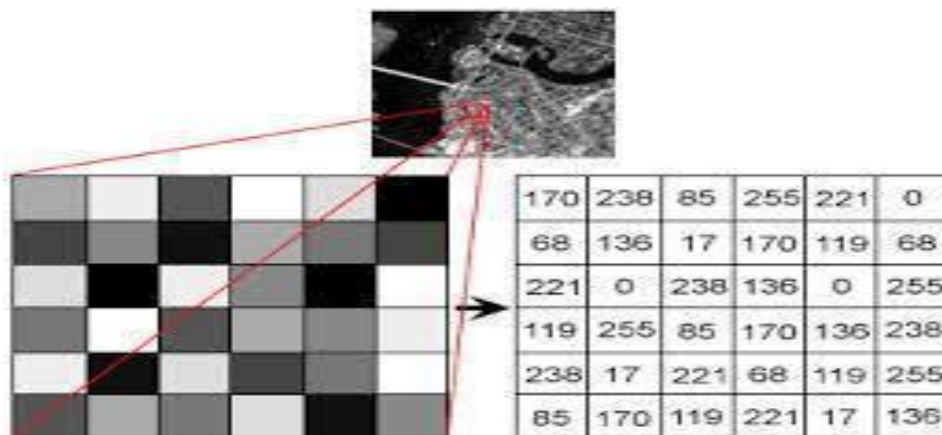


Fig 2.2 Effect of digitization

Image pre-processing :

1. Scaling :

The theme of the technique of magnification is to have a closer view by magnifying or zooming the interested part in the imagery. By reduction , we can bring the unmanageable size of data to a manageable limit. For resampling an image, Nearest Neighbourhood, Linear, or cubic convolution techniques are used.

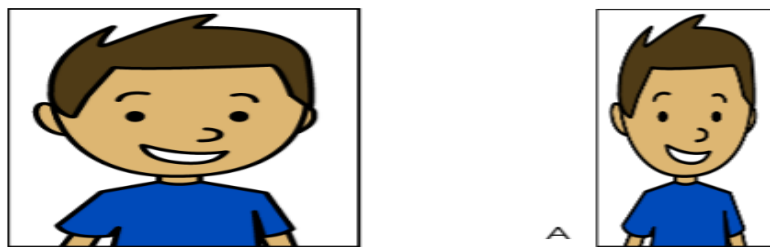


Fig 2.3 Scaling

2. Magnification :

This is usually done to improve the scale of display for visual interpretation or sometimes to match the scale of one image to another. To magnify an image by a factor of 2, each pixel of the original image is replaced by a block of 2x2 pixels, all with the same brightness value as the original pixel .



Fig 2.4 Magnification

3. Reduction :

To reduce a digital image to the original data, every mth and nth column of the original imagery is selected and displayed. Another way of accomplishing the same is by taking the average in 'm x m' block and displaying this average after proper rounding of the resultant value.

4. Rotation :

Rotation is used in image mosaics, image registration etc. One of the techniques of rotation is 3-pass shear rotation, where a rotation matrix can be decomposed into three separable matrices.

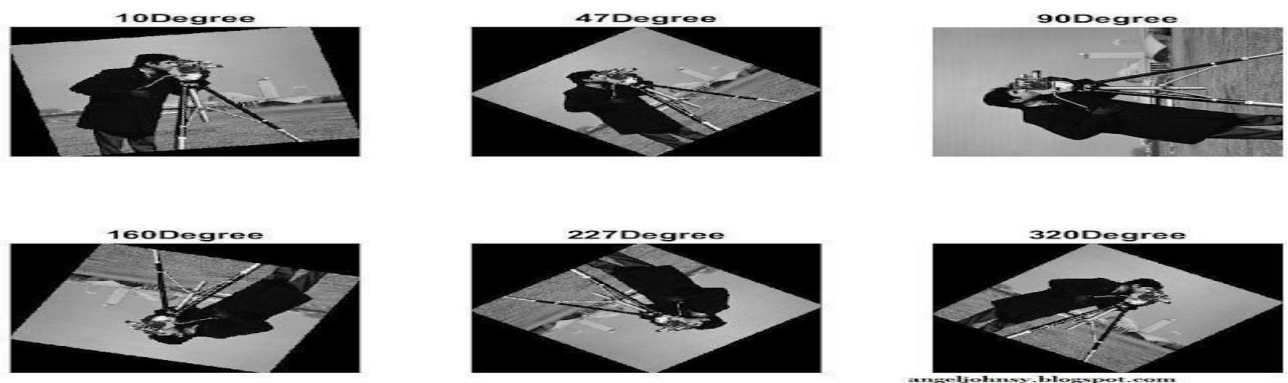


Fig 2.5 Rotation

5. Mosaic :

Mosaic is a process of combining two or more images to form a single large image without radiometric imbalance. Mosaic is required to get the synoptic view of the entire area, otherwise captured as small images.

6. Contrast stretching :

Some Images dense forests, snow, clouds and under lazy conditions over heterogeneous regions)are homogeneous i.e., they do not have much change in their levels. In terms of histogram representation, they are characterised as the occurrence of

very narrow peaks. The homogeneity can also be due to the incorrect illumination of the scene.

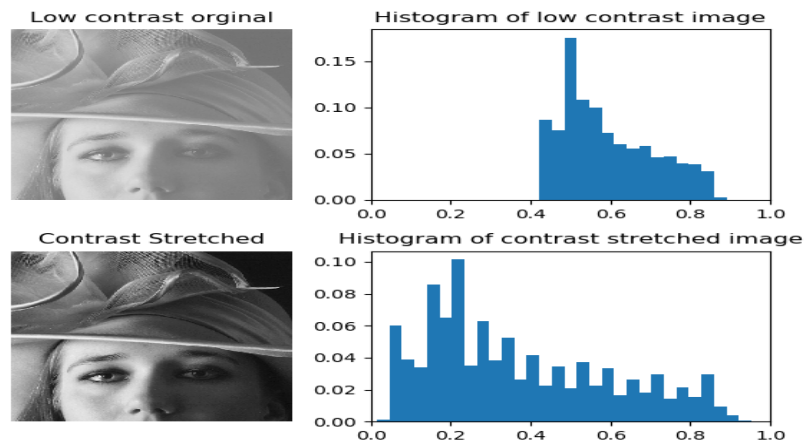


Fig 2.6 Contrast Stretching

7. Noise Filtering :

Noise filtering is used to filter the unnecessary information from an image. It is also used to remove various types of noises from the images. Mostly this feature is interactive. Various filters like low pass, high pass , mean, median etc., are available.



Fig 2.7 Noise Filtering

2.3 ADVANTAGES:

Digital image processing in the most layman terms is image editing to improve its visual appearance but not limited to it. The main advantages of digital image processing are

1. Digital images can be processed by digital computers.
2. Important features such as edges can be extracted from images which can be
In industry
3. Images can be given more sharpness and better visual appearance.
4. Minor errors can be rectified.
5. Image sizes can be increased or decreased.
6. Images can be compressed and decompressed for faster image transfer over the network

2.4 APPLICATIONS :

- Computerised photography (e.g., Photoshop)
- Space image processing (e.g., Hubble space telescope images, interplanetary probe images)
- Medical/Biological image processing (e.g., interpretation of X-ray images, blood/cellular microscope images)
- Automatic character recognition (zip code, licence plate recognition)
- Fingerprint/face/iris recognition
- Remote sensing: aerial and satellite image interpretations

CHAPTER -3

IMAGE FUSION

CHAPTER-3

IMAGE FUSION

3.1 INTRODUCTION:

Image fusion is the process by which two or more images are combined into a single image retaining the important features from each of the original images. The fusion of images is often required for images acquired from different instrument modalities or capture techniques of the same scene or objects. Fusion techniques include the simplest method of pixel averaging to more complicated methods such as principal component analysis and wavelet transform fusion.

Several approaches to image fusion can be distinguished, depending on whether the images are fused in the spatial domain, or they are transformed into another domain, and their transforms fused. The successful fusion of images acquired from different modalities or instruments is of great importance in many applications such as medical imaging, microscopic imaging, remote sensing computer vision and robotics. Many methods exist to perform image fusion. Image fusion can be defined as the process by which several images or some of their features are combined to form a single image fusion that can be performed at different levels of the information representation. Four different levels can be distinguished, i.e., signal pixel feature.

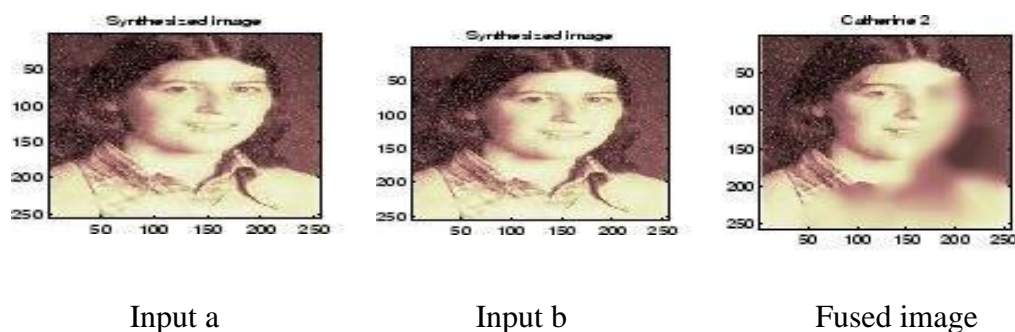


Fig 3.1 Example for image fusion

To date the results of image fusion in areas such as remote sensing and medical imaging are primarily intended for presentation to a human observer for easier and

enhanced interpretation. Therefore, the perception of the fused image is of paramount importance when evaluating different fusion schemes. Some generic requirements can be imposed on the fusion result. The fused image should preserve as closely as possible all relevant information contained in the input images. The fusion process should not introduce any artifacts or inconsistencies which can distract or mislead the human observer or any subsequent image processing steps. In the fused image, relevant features and noise should be suppressed to a maximum extent. Spatial distortion can be very well handled by frequency domain approaches on image fusion. Multi-resolution analysis has become a very useful tool for analysing remote sensing images. The discrete wavelet transform has become a very useful tool for fusion. Some other fusion methods are also there such as Laplacian- pyramid based, Curvelet transform based etc. These methods show a better performance in spatial and spectral quality of the fused image compared to other spatial methods of fusion.

Benefits of image fusion:

- Image overlay for displays
- Image sharpening for operator clarity
- Image enhancement through noise reduction
- Image mosaicking for enhanced spatial coverage
- Image registration for reference to world coordinates
- Enhanced clarity through feature amplification

Image fusion fuses two or more images and synthesises them into one that contains all the significant or clear information from each input image. These images may be acquired from different sensing devices, or they may be of the same scene with focus on different parts of it. These include spatial and spectral resolution, quantity of information and details of features of interest. So, the image fusion can be divided into three types:

1. Pixel Level Fusion
2. Feature level fusion
3. Decision level fusion

Pixel level fusion:

Pixel-level image fusion is the information fusion that is implemented directly using the basic data of the images needed to be fused. It is image fusion at the lowest level that has the highest degree of integration accuracy. Pixel-level fusion is able to provide the detailed information of images that cannot be brought by those at the other levels. Of course, it should

be noted that pixel-level fusion requires the largest amount of information. Its structure is shown in Fig1.1. It is worthy to be emphasised that pixel-level image fusion integrates the information of multi-source images on the premise of strict registration.

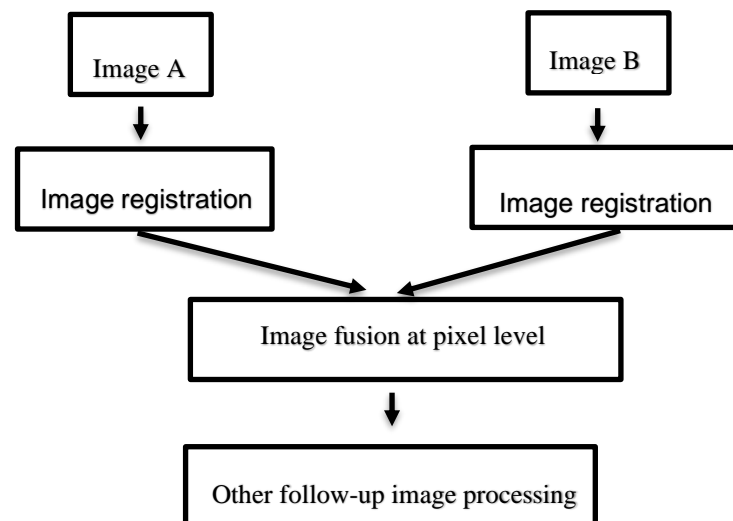


Fig 3.2 Pixel level fusion

Feature level fusion:

Feature-level fusion belongs to the information integration processed at the middle level. It not only retains sufficient important information, but also compresses the information. So it is beneficial to real-time processing. In fact; the features extracted from multi source images are synthesised and processed in feature-level fusion.

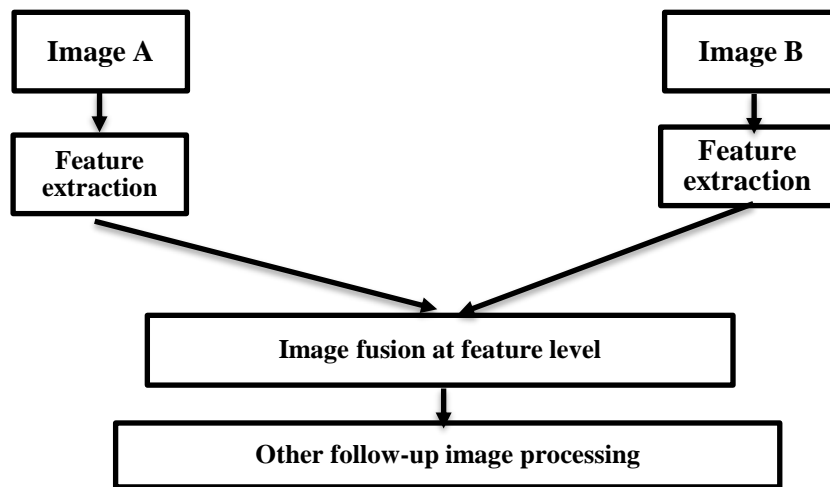


Fig 3.3 Feature level fusion

Decision level data fusion:

It is a multi-level, multifaceted process handling the automatic detection, association, correlation, estimation, and combination of data from several sources. Its objective is to make the optimal or suboptimal use of the information generated by multiple knowledge sensors.

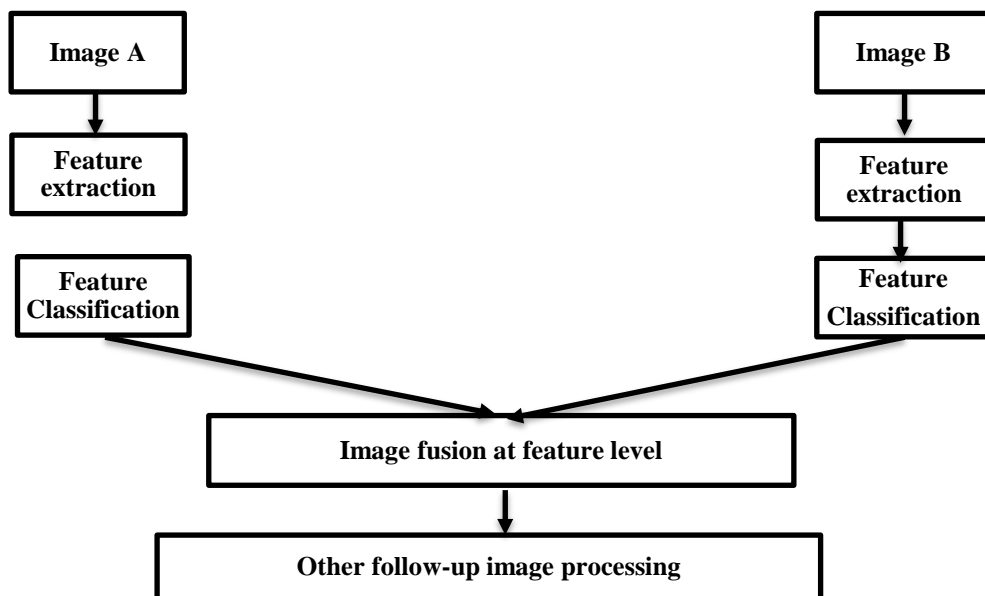


Fig 3.4 Decision level fusion

3.2 NEED FOR IMAGE FUSION:

Multi-sensor data fusion has become a discipline which demands more general formal solutions to a number of application cases. Several situations in image processing require both high spatial and high spectral information in a single image. This is important in remote sensing. However, the instruments are not capable of providing such information either by design or because of observational constraints. One possible solution for this is data fusion. The fusion of SAR image and optical image can be used for remote sensing applications much more easily and accurately. Many image fusion algorithms and devices have shown notable achievements in improving decisions based on multi-source remote sensing images .There is a growing interest and application of the imaging technologies in the areas of medical diagnostics, geographical analysis and historical documentation. Since computer aided imaging techniques enable a quantitative assessment of the images under evaluation, it helps to improve the efficacy of the medical practitioners in arriving at an unbiased and objective decision in a short span of time. In addition, the use of multi-sensors .

3.3 IMAGE FUSION TYPES:

- 1.Multiview image fusion
2. Multimodal image fusion
- 3.Multi Temporal image fusion
- 4.Multifocus image fusion
- 5.Fusion for image restoration

Multiview Fusion:

- Images of the same modality, taken at the same time but from different places or under different conditions
- Goal: to supply complementary information from different views.

Multimodal Fusion:

Images of different modalities like visible, infrared, ultraviolet, etc are used in this type of fusion. The goal of multimodal fusion is to decrease the amount of data, to emphasise band-specific info.

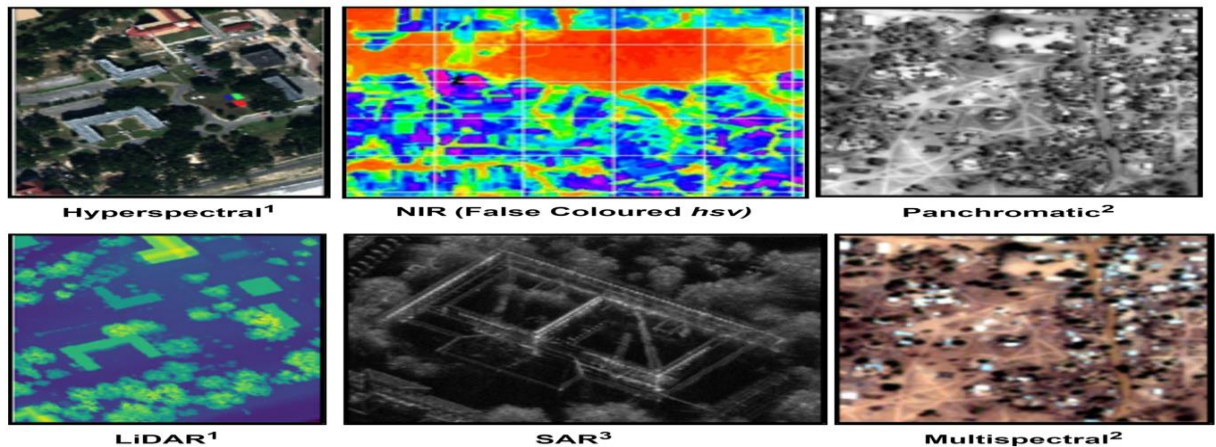


Fig 3.5 Multimodal fusion

Multimodal Fusion Common methods:

- Weighted averaging pixel-wise
- Fusion in transform domains
- Object-level fusion

Multitemporal Fusion:

Images of the same scene taken at different times, usually of the same modality, are used in this type of fusion process. The main objective of this fusion method is to detect the changes in the images.

Common Methods are subtraction.

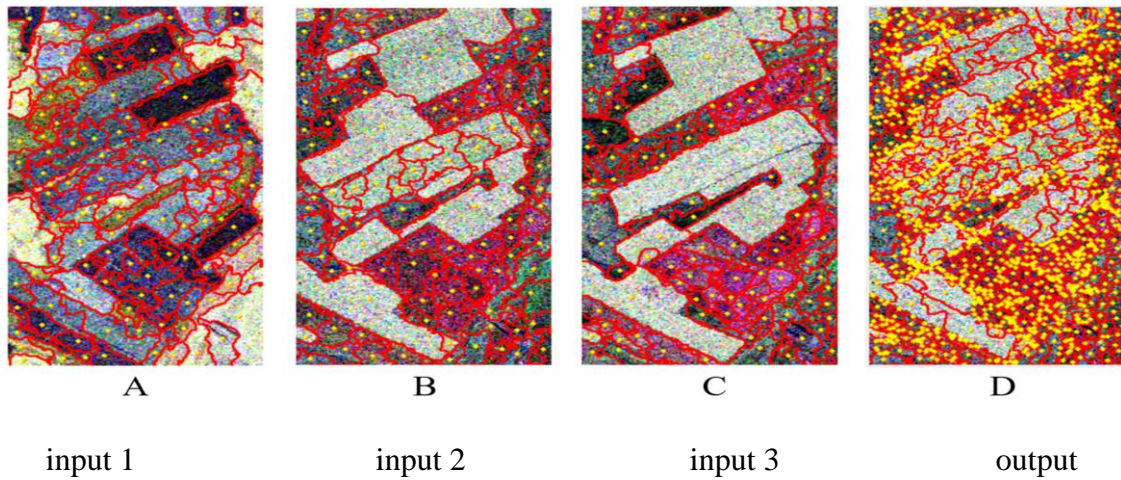


Fig 3.6 Multitemporal fusion

Multifocus fusion:

The original image can be divided into regions such that every region is in focus in at least one channel and can be used for the fusion process. The main goal is to focus the image everywhere.

Common Method is identifying the regions in focus and combine them

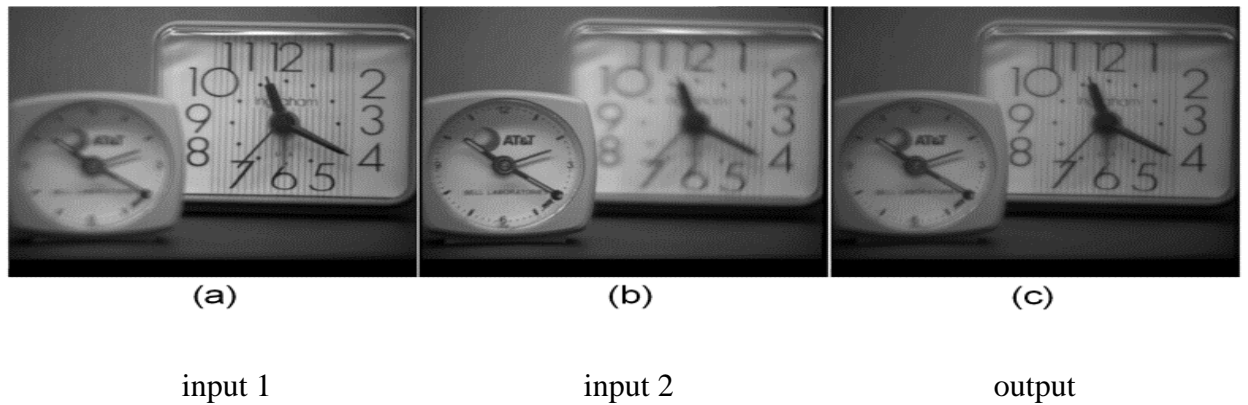


Fig 3.7 Multifocus fusion

Fusion for image restoration:

Each image consists of the “true” part and the “degradation” part. This degradation part is now removed by the fusion method.

Types of degradation:

- additive noise: image denoising
- convolution: blind deconvolution
- resolution decimation: super resolution

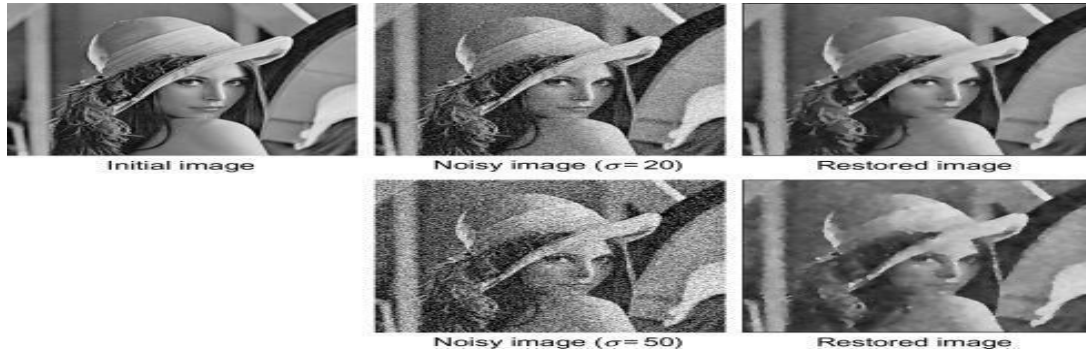


Fig 3.8 Fusion for Image Restoration

3.4 IMAGE FUSION TECHNIQUES:

There are various methods that have been developed to perform image fusion. Some well-known image fusion methods are listed below:

- Averaging method
- Select Maximum method
- Select Minimum method
- Principal Component Analysis Method
- Filter Subtract Decimate Pyramid Method
- Gradient Pyramid Method
- Ratio Pyramid Method
- Morphological Pyramid Method
- Laplacian Pyramid Method
- Haar Wavelet Method
- DBSS (2,2) wavelet Method

Averaging Method:

The very concept of information fusion arose from the idea of averaging the available information. In this method the basic concept to fuse a set of input images was to average the pixel intensities of the corresponding pixels. The fused image produced by this method projects both the good and the bad information from the input images. Thus the algorithm does not actually fuse the images perfectly. The value of the pixel $P(i, j)$ of each image is taken and added. This sum is then divided by 2 to obtain the average. The average value is assigned to the corresponding pixel of the output image which is given in the equation .1

$$K(i, j) = \{X(i, j) + Y(i, j)\} / 2 \quad (1) \text{ Where } X(i, j) \text{ and } Y(i, j) \text{ are two input images.}$$

Select Maximum:

In this method, the pixel with maximum intensity is selected and is put in as the resultant pixel of the fused image. Thus, effectively, every pixel of the fused image will be the pixel with maximum intensity of the corresponding position pixels in the input image. One advantage of this method over averaging is that there is no compromise made over the good information available in the input images. It depends on the type of image under consideration. Thus, you either take the whole of the information or totally avoid the same. The greatest pixel value is assigned to the corresponding pixel of the resultant fused image.

Select Minimum:

The minimum selecting method, being yet another trivial image fusion method, is very similar to the Maximum Selection method; except for, here, the selection criteria differ as the pixel with minimum density is picked up. Thus, for every pixel position, the pixel of the fused image will be the pixel of the corresponding position from the input set of images having the lowest pixel intensity value. The quality of the fusion is specific to the type of image we are dealing with.

Principal Component Analysis:

The PCA is used extensively in image compression and image classification. The PCA involves a mathematical procedure that transforms a number of correlated variables into a number of uncorrelated variables called principal components. It computes a compact and optimal description of the data set. The PCA algorithm looks into scaling the pixel values of the images with a weight. The algorithm can be summarised as the following :

1. Generate the column vectors, respectively, from the input image matrices.
2. Calculate the covariance matrix of the two column vectors formed in 1.
3. The diagonal elements of the 2x2 covariance vector would contain the variance of each column vector with itself, respectively.
4. Calculate the Eigenvalues and the Eigenvectors of the covariance matrix.
5. Normalise the column vector corresponding to the larger Eigenvalue by dividing each element with the mean of the Eigenvector.
6. Sum of the two scaled matrices calculated in 5 will be the fused image matrix.

Filter Subtract Decimate Pyramid Method:

Every pyramid transform consists of two major phases: Decomposition, Formation of the initial image for decomposition.

Decomposition is the process where a pyramid is generated successively at each level of the fusion. The depth of fusion or number of levels of fusion is pre decided. Image Fusion Methods and Quality Assessment Parameters .The number of levels of fusion is decided based on the size of the input image. In this method the decomposition phase consists of three steps :

- Using Low pass filtering .
- Subtract the low pass filtered input images and form the pyramid.
- Decimate the input image matrices by halving the number of rows and columns (we did by neglecting every alternate row and column).

The decomposition phase would include steps:

- Decimating the image matrix by duplicating every row and column

- Low pass filtering with $2*W$
- Matrix addition is the same as the pyramid formed in the corresponding level.

Laplacian Pyramid Method:

The Laplacian pyramidal method is identical to FSD pyramid except for an additional low pass filtering performed with $2*W$. All the other steps are followed as in the FSD pyramid.

Ratio Pyramid Method:

The Ration pyramidal method is also identical to FSD pyramid except for, in the decomposition phase, after low pass filtering the input image matrices, the pixel wise ratio is calculated instead of subtraction as in FSD.

Gradient Pyramid Method:

The decomposition process here would include the following steps: Two low pass filters are considered here Additional to this; four directional filters are applied on to the input image matrices.

Morphological Pyramid Method:

The decomposition phase in this method consists of the following steps:

Two levels of filtering are performed on the input image matrices. Image opening and image closing. followed by image dilation. Image closing is the other way round. A combination of image opening, and image closing gets rid of noise in the image and the rest of the steps are as in the FSD pyramid method.

Haar Wavelet Method:

The Haar wavelet is the first known wavelet. The Haar wavelet $\psi(t)$ can be described

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

and its scaling function $\varphi(t)$ can be described as

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

The 2x2 Haar matrix is associated with the filters thus, considered here would be

$$F1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F2 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

The couple of filters, when applied on the input images matrices, would produce 4 resultant matrices. The fourth matrix, which would consist of all the high frequencies, would act as the input matrix for the next level of decomposition. The other three matrices, consisting of the low frequencies, are used to produce 3 pyramids at each level. The pyramids are produced as a pyramidal method. The decomposition process is performed with the help of the three pyramids formed at each level of decomposition. The disadvantage of the Haar wavelet is that it is not continuous and therefore not differentiable.

DBSS (2,2) Wavelet Method:

The Daubechies wavelets are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support in general the Daubechies wavelets are chosen to have the highest number A of vanishing moments, (this does not imply the best smoothness) for given support width $N=2A$, and among the $2A-1$ possible solutions the one is chosen whose scaling filter has extremal phase.

CHAPTER -4
WAVELET and GRAM-SCHMIDT
TRANSFORM

CHAPTER - 4

WAVELET and GRAM-SCHMIDT TRANSFORM

4.1 INTRODUCTION

A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualised as a "brief oscillation" like one recorded by a seismograph or heart monitor. Generally, wavelets are intentionally crafted to have specific properties that make them useful for signal processing. Using a "reverse, shift, multiply and integrate" technique called convolution, wavelets can be combined with known portions of a damaged signal to extract information from the unknown portions.

For example, a wavelet could be created to have a frequency of Middle C and a short duration of roughly a 32nd note. If this wavelet were to be convolved with a signal created from the recording of a song, then the resulting signal would be useful for determining when the Middle C note was being played in the song. Mathematically, the wavelet will correlate with the signal if the unknown signal contains information of similar frequency. This concept of correlation is at the core of many practical applications of wavelet theory.

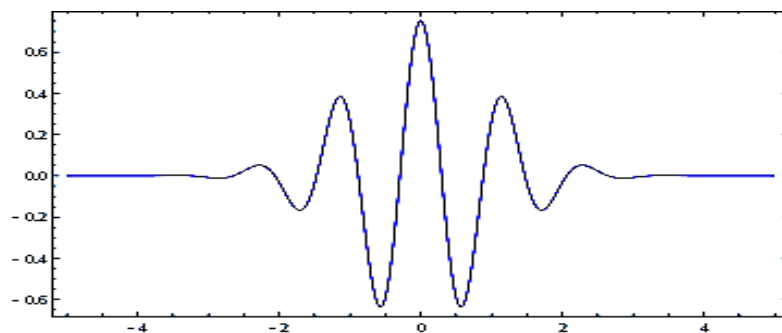


Fig 4.1 Example of a wavelet

As a mathematical tool, wavelets can be used to extract information from many kinds of data, including – but certainly not limited to – audio signals and images. Sets of wavelets are generally needed to analyse data fully. A set of "complementary" wavelets

will decompose data without gaps or overlap so that the decomposition process is mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet-based compression/decompression algorithms where it is desirable to recover the original information with minimal loss.

DEFINITION OF WAVELET:

The wavelet can be defined in different ways :

Scaling filter:

an orthogonal wavelet is entirely defined by the scaling filter – a low-pass finite impulse response (FIR) filter of length $2N$ and sum 1. In biorthogonal wavelets, separate decomposition and reconstruction filters are defined.

For analysis with orthogonal wavelets the high pass filter is calculated as the quadrature mirror filter of the low pass, and reconstruction filters are the time reverse of the decomposition filters.

E.g., Daubechies and Symlet wavelets can be defined by the scaling filter.

Scaling function:

Wavelets are defined by the wavelet function $\psi(t)$ (i.e. the mother wavelet) and scaling function $\phi(t)$ (also called father wavelet) in the time domain.

The wavelet function is in effect a band-pass filter and scaling it for each level halves its bandwidth. This creates the problem that in order to cover the entire spectrum, an infinite number of levels would be required. The scaling function filters the lowest level of the transform and ensures all the spectrum is covered.

For a wavelet with compact support, $\phi(t)$ can be considered finite in length and is equivalent to the scaling filter g .

E.g., Meyer wavelets can be defined by scaling functions

Wavelet function:

The wavelet only has a time domain representation as the wavelet function $\psi(t)$.

E.g., Mexican hat wavelets are defined by wavelet function.

4.2 DIFFERENT TYPES OF WAVELETS:

The wavelets are classified as:

- Discrete wavelets
- Continuous wavelets
- Real valued wavelet
- Complex valued wavelet

Discrete wavelets :

Haar wavelet :

In mathematics, the Haar wavelet is a sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal basis. The Haar sequence is now recognised as the first known wavelet basis and extensively used as a teaching example.

The Haar sequence was proposed in 1909 by Alfréd Haar. Haar used these functions to give an example of an orthonormal system for the space of square-integrable functions on the unit interval $[0, 1]$. The study of wavelets, and even the term "wavelet", did not come until much later. As a special case of the Daubechies wavelet, the Haar wavelet is also known as **Db1**.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines.

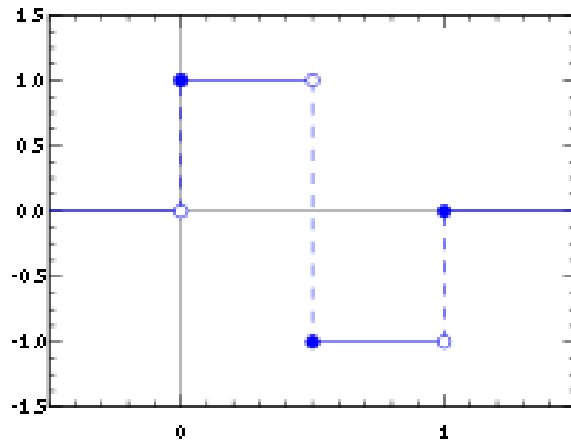


Fig 4.2 Haar wavelet function

The Haar wavelet mother wavelet function is defined as

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Its scaling function is described as:

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Daubechies wavelet :

The **Daubechies wavelets**, based on the work of Ingrid Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterised by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis.

In general, the Daubechies wavelets are chosen to have the highest number A of vanishing moments, (this does not imply the best smoothness) for given support width $2A - 1$. There are two naming schemes in use, DN using the length or number of taps, and dbA referring to the number of vanishing moments. So D4 and db2 are the same wavelet transform.

The Daubechies wavelets are not defined in terms of the resulting scaling and wavelet functions; in fact, they are not possible to write down in closed form. The graphs below are generated using the cascade algorithm, a numeric technique consisting of simply inverse transforming [1 0 0 0 ...] an appropriate number of times.

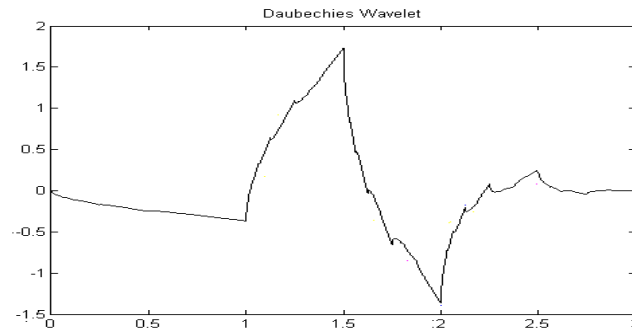


Fig4.3 Daubechies wavelet

CONTINUOUS WAVELET:

Mexican Hat Wavelet:

$$\varphi(t) = \frac{2}{\sqrt{3\sigma\pi^{0.25}}} \left(1 - \left(\frac{t}{\sigma}\right)^2\right) e^{-\frac{t^2}{2\sigma^2}}$$

The ricker wavelet is the **negative** normalised **second** derivative of a Gaussian function, i.e., up to scale and normalisation, the second Hermite function. It is a special case of the family of continuous wavelets (wavelets used in a continuous wavelet transform) known as Hermitian wavelets. The Ricker wavelet is frequently employed to model seismic data, and as a broad-spectrum source term in computational electrodynamics. It is usually only referred to as the **Mexican hat wavelet** in the Americas, due to taking the shape of a sombrero when used as a 2D image processing kernel. It is also known as the **Marr wavelet** for David Marr.

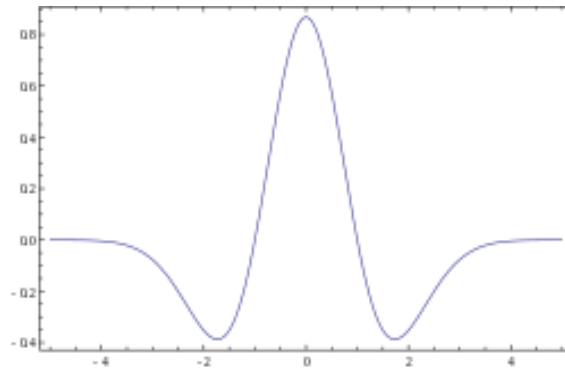


Fig 4.4 Mexican Hat wavelet

4.3 WAVELET TRANSFORMS:

A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually, one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale. A wavelet transform is the representation of a function by wavelets. The wavelets are scaled and translated copies (known as "daughter wavelets") of a finite-length or fast-decaying oscillating waveform (known as the "mother wavelet"). Wavelet transforms have advantages over traditional Fourier transforms for representing functions that have discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.

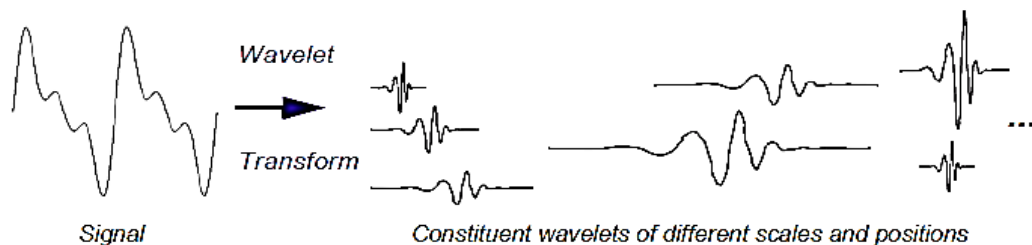


Fig 4. 5 Representation of signal in different scales and positions

DEFINITION:

A function $\varphi \in L^2(\mathbb{R})$ is called an orthonormal wavelet if it can be used to define a Hilbert basis, that is a complete orthonormal system, for the Hilbert space $L^2(\mathbb{R})$ under the standard inner product on $L^2(\mathbb{R})$, this family is orthonormal, it is an orthonormal system:

$$\langle \varphi_{jk}, \varphi_{lm} \rangle = \int_{-\infty}^{\infty} \varphi_{jk}(x) \overline{\varphi_{lm}(x)} dx = \delta_{jk} \delta_{lm}$$

where δ_{jl} is the Kronecker delta.

Completeness is satisfied if every function $f \in L^2(\mathbb{R})$ may be expanded in the basis as

$$f(x) = \sum_{j,k=-\infty}^{\infty} c_{jk} \varphi_{jk}(x)$$

with convergence of the series understood to be convergence in norm. Such a representation of f is known as wavelet series. This implies that an orthonormal wavelet is self dual.

The integral wavelet transform is the integral transform defined as

$$[W_{\varphi} f](a, b) = \frac{1}{\sqrt{|a|}}$$

The wavelet coefficients c_{jk} are then given by

$$c_{jk} = [W_{\varphi} f](2^{-j}k, 2^{-j})$$

Here $a = 2^{-j}$ is called the binary dilation or dyadic dilation and $b = k2^{-j}$ is called the binary or dyadic position.

PRINCIPLE

The fundamental idea of wavelet transforms is that the transformation should allow only changes in time extension, but not shape. This is affected by choosing suitable basis functions that allow for this. Changes in the time extension are expected

to conform to the corresponding analysis frequency of the basis function. Based on the uncertainty principle of signal processing,

$$\Delta t \cdot \Delta \omega \geq 1/2$$

where t represents time and ω angular frequency ($\omega = 2\pi f$, where f is temporal frequency).

The higher the required resolution in time, the lower the resolution in frequency has to be. The larger the extension of the analysis windows is chosen, the larger is the value of Δt .

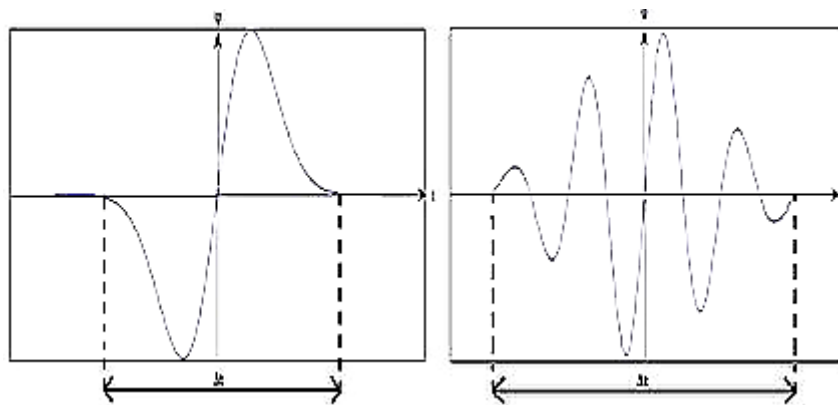


Fig4.6 illustration of short time Fourier transform

When Δt is large,

1. Bad time resolution
2. Good frequency resolution
3. Low frequency, large scaling factor

When Δt is small

1. Good time resolution
2. Bad frequency resolution
3. High frequency, small scaling factor

In other words, the basis function Ψ can be regarded as an impulse response of a system with which the function $x(t)$ has been filtered. The transformed signal provides information about the time and the frequency. Therefore, wavelet-transformation contains information like the short-time-Fourier-transformation, but with additional special properties of the wavelets, which show up at the resolution in time at higher analysis frequencies of the basis function. The difference in time resolution at ascending frequencies for the Fourier transform and the wavelet transform is shown below.

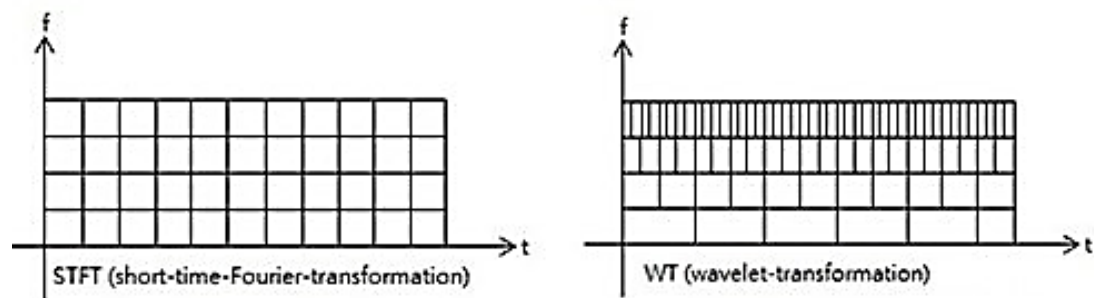


Fig 4.7 Time resolution of STFT and WT

This shows that wavelet transformation is good in time resolution of high frequencies, while for slowly varying functions, the frequency resolution is remarkable.

4.4 TYPES OF WAVELET TRANSFORMS

There are different types of wavelet transforms. Some of them are:

1. Continuous wavelet transform
2. Discrete wavelet transform'
3. Stationary wavelet transform
4. Complex wavelet transform
5. Dual tree complex wavelet transform
6. Lifting wavelet transform

4.4.1 DISCRETE WAVELET TRANSFORM:

It turns out, rather remarkably, that if we choose scales and positions based on powers of two so-called dyadic scales and positions. Then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform (DWT).

$$w(j, k) = f(t)$$

Here \mathbf{j}, \mathbf{k} are integers representing the set of discrete translations and discrete dilations an efficient way to implement this scheme using filters was developed in 1988 by Mallat. The Mallat algorithm is in fact a classical scheme known in the signal processing community as a two-channel sub band. This very practical filtering algorithm yields a fast wavelet transform a box into which a signal passes, and out of which wavelet coefficients quickly emerge

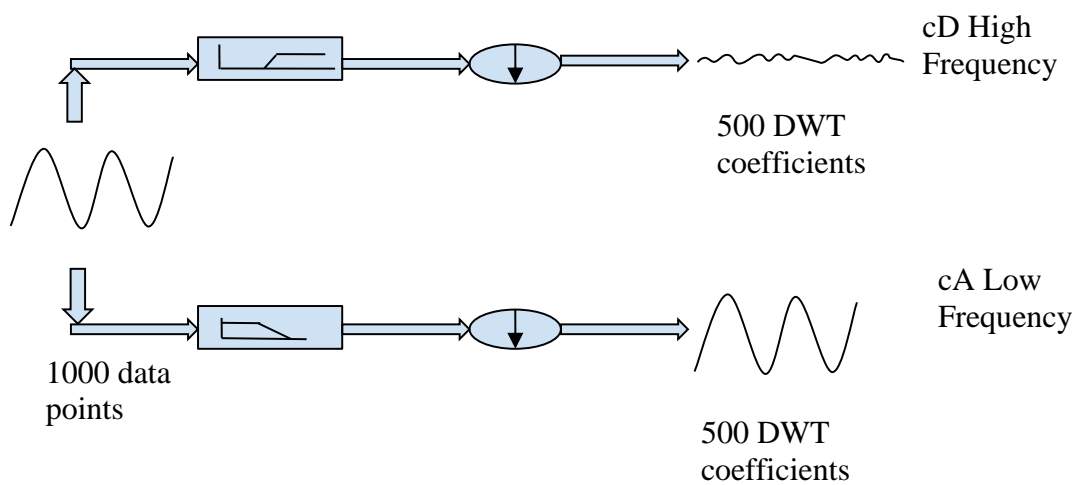
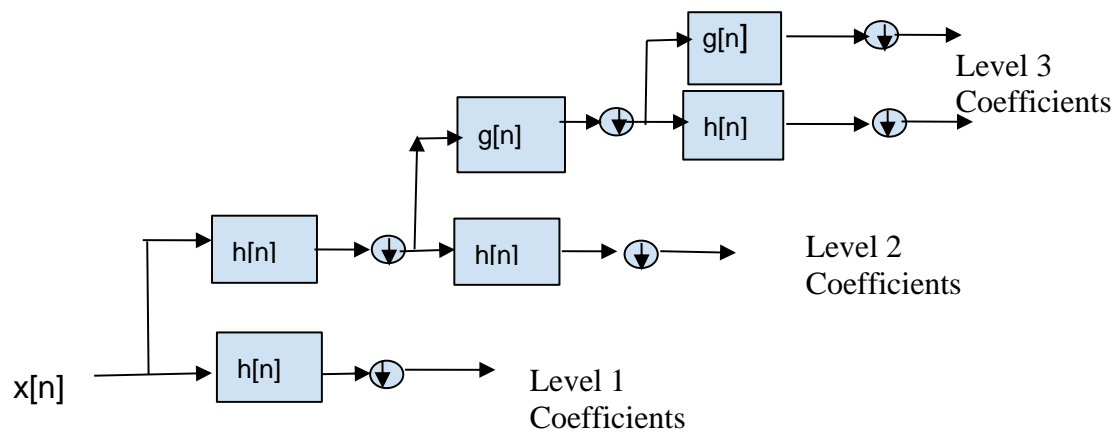


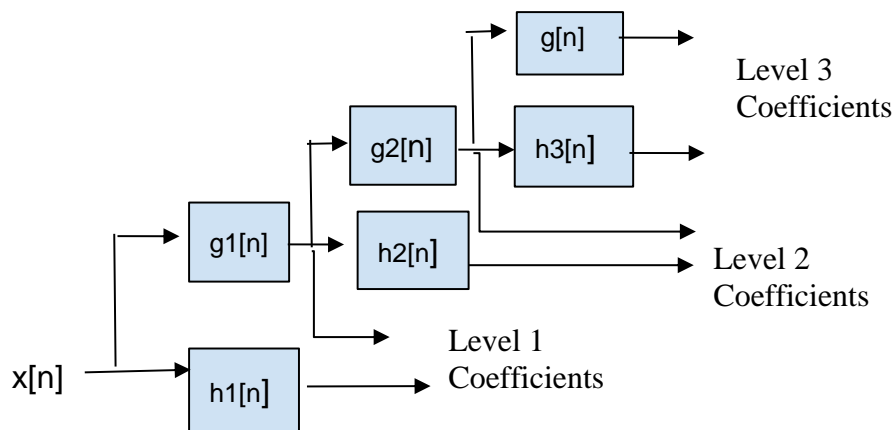
Fig. 4.8 Discrete wavelet transform

Figure 4.7 shows that discrete wavelet transform represents discrete wavelet coefficients and which contains the higher frequency, lower frequency coefficients. A critical sampling defines the resolution of the DWT in both time and frequency. For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavour or

nuance. The process on the right, which includes down sampling, produces DWT coefficients. The detail coefficients cD are small and consist mainly of a high-frequency noise, while the approximation coefficients cA contain much less noise than does the original signal. The block diagram of DWT is shown below:



(a)



(b)

Fig 4.9 Block diagram of DWT

We can use the term critical sampling to denote the minimum number of coefficients sampled. Critical sampling defines the resolution of the DWT in both time and frequency. In critical sampling of the time-frequency plane by discretizing parameter Shown in figure 4.8

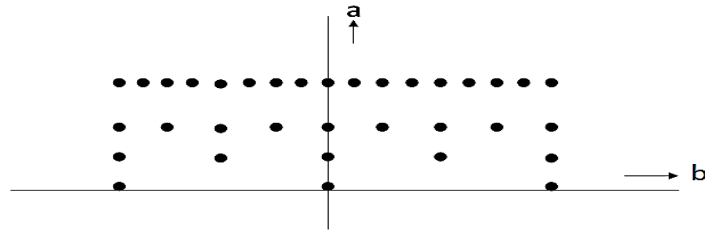


Fig.4.10 Time and frequency plane of the wavelet transform.

4.4.2 DUAL TREE COMPLEX WAVELET TRANSFORMS:

The dual tree complex wavelet transform is a relatively recent enhancement to the discrete wavelet transform with important additional properties. It is nearly shift-invariant and directionally selective in two and higher dimensions. It achieves this with a redundancy factor of only 2^d for the d -dimension signal, which is substantially lower than undecimated DWT. The multidimensional dual tree CWT is non-separable but is based on a computationally efficient, separable filter bank (FB).

The dual tree CWT employs two real DWTs; the first DWT gives the real part of the transform while the second DWT gives the imaginary part. The analysis and synthesis FBs used to implement the dual-tree CWT and its inverse are illustrated. The two real wavelet transforms use two different sets of filters, with each satisfying the PR conditions. The two sets of filters are jointly designed so that the overall transform is approximately analytic. Let $h_0(n)$, $h_1(n)$ denote the low-pass/high-pass filter pair for the upper FB, and let $g_0(n)$, $g_1(n)$ denote the low-pass/high-pass filter pair for the lower FB. We will denote the two real wavelets associated with each of the two real wavelet transforms as $\varphi_h(t)$ and $\varphi_g(t)$. In addition to satisfying the PR conditions, the filters are designed so that the complex wavelet $\varphi(t) = \varphi_h(t) + j\varphi_g(t)$ is approximately analytic. Equivalently, they are designed so that $\varphi_g(t)$ is approximately the Hilbert transform of $\varphi_h(t)$.

Note that the filters are themselves real; no complex arithmetic is required for the implementation of the dual tree. Also note that the dual-tree CWT is not a critically

sampled transform; it is two times expansive in 1-D because the total output data rate is exactly twice the input data rate.

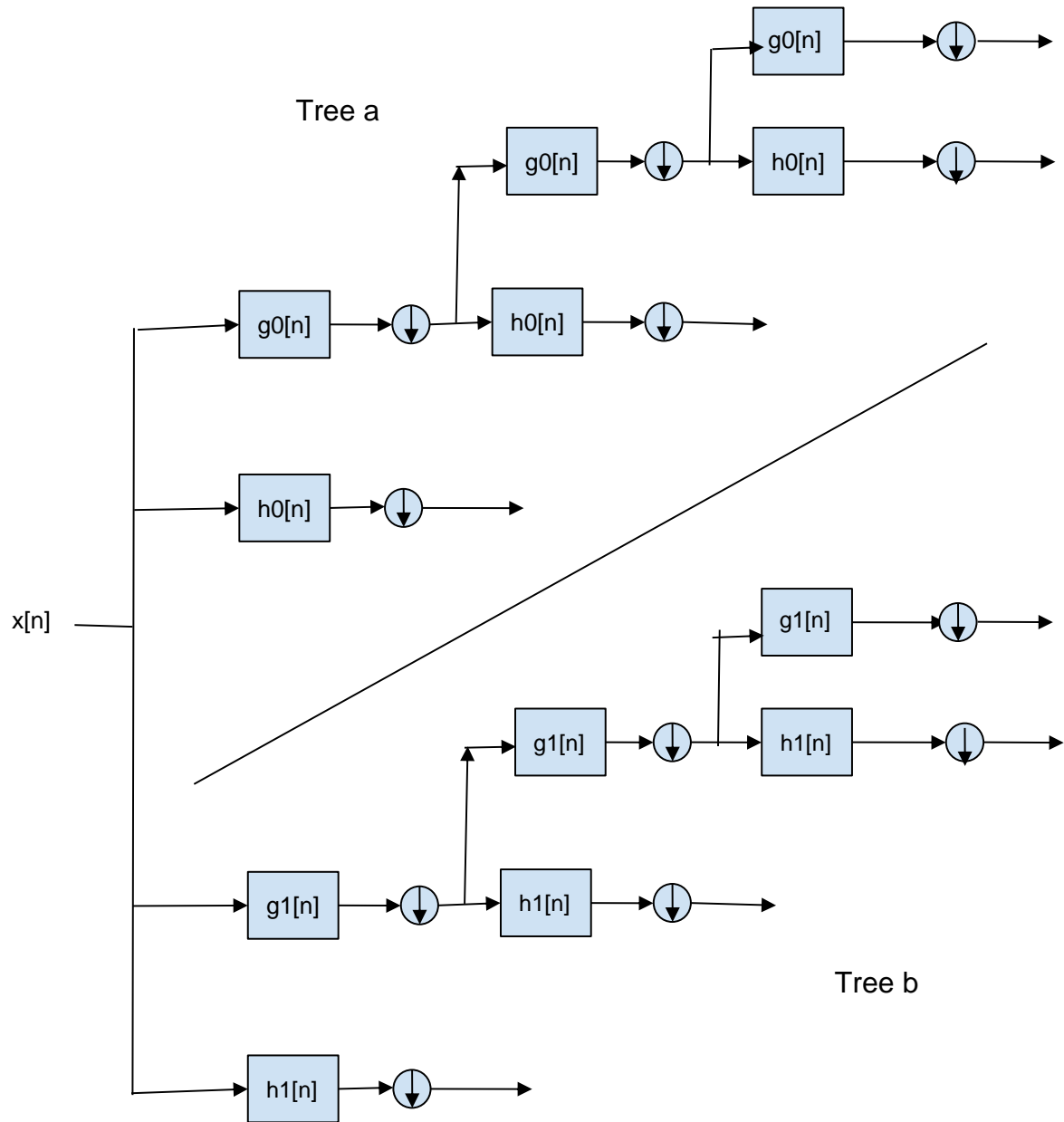


Fig 4.11 Block diagram of DT-CWT

The inverse of the dual-tree CWT is as simple as the forward transform. To invert the transform, the real part and the imaginary part are each inverted—the inverse of each of the two real DWTs are used—to obtain two real signals. These two real signals are then averaged to obtain the final output. Note that the original signal $x(n)$

can be recovered from either the real part or the imaginary part alone; however, such inverse dual-tree CWTs do not capture all the advantages an analytic wavelet transform offers.

If the two real DWTs are represented by the square matrices F_h and F_g then the dual-tree CWT can be represented by the rectangular matrix

$$F = [F_h F_g]$$

If the vector X represents a real signal, then $w_h = F_h \cdot x$ represents the real part and $w_g = F_g \cdot x$ represents the imaginary part of the dual tree CWT.

$$F^{-1} = \frac{1}{2} [F_h^{-1} F_g^{-1}]$$

If the two real DWTs are orthonormal transforms, then the transpose of F_h is its inverse $F_h^t \cdot F_h = I$ and similarly for F_g . In this case, the transpose of the rectangular matrix F is also a left inverse $F^t \cdot F = I$, where we have used \cdot . That is, the inverse of the dual-tree CWT can be performed using the transpose of the forward dual-tree CWT; it is self-inverting .

The dual-tree wavelet transform defined in (6) keeps the real and imaginary parts of the complex wavelet coefficients separate. If

$$[vu] = F_c \cdot x$$

then when x is real, we have $v = u^*$, so v need not be computed. When the input signal x is complex, then $v \neq u^*$, so both u and v need to be computed.

4.4.3 CONTINUOUS WAVELET TRANSFORM:

Mathematically, the process of Fourier analysis is represented by the Fourier transform:

$$f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Which is the sum over all time of the signal $f(t)$ multiplied by a complex exponential. (Recall that a complex exponential can be broken down into real and imaginary sinusoidal components.

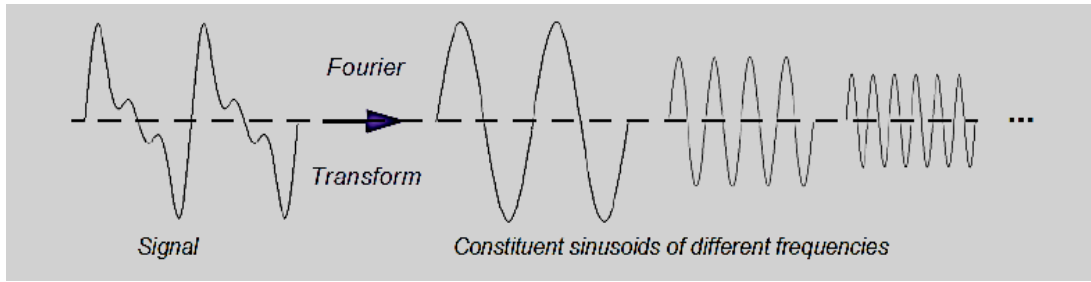


Fig.4.12 Continuous wavelet transform of a signal

The transform are the Fourier coefficients $F(\omega)$, which when multiplied by a sinusoid of frequency ω yield the constituent sinusoidal components of the original signal shows in the figure 2.8 similarly, the continuous wavelet transform (CWT) is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function Ψ

$$C(\text{scale}, \text{position}) = \int_{-\infty}^{\infty} f(t)\psi(\text{scale}, \text{position}, t)dt$$

The results of the CWT are many wavelets coefficients C , which are a function of scale and position. Discretization of the CWT through sampling specific wavelet coefficients. A critical sampling of the CWT

$$w(a, b) = \int x(t) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) dt$$

$$\Psi_{a, b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

$$\psi\left(\frac{t-b}{a}\right) \Psi_{a, b}(t)$$

Here $x(t)$ is transferred by an analysing function $\psi\left(\frac{t-b}{a}\right) \Psi_{a, b}(t)$

a scaling parameter

b time shift parameter

Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal.

4.4.4. STATIONARY WAVELET TRANSFORM :

The SWT provides efficient numerical solutions in signal processing applications. It was independently developed by several researchers and under different names, e.g. the undecimated wavelet transform, the invariant wavelet transform and the redundant wavelet transform. The key point is that it gives a better approximation than the discrete wavelet transform (DWT) since it is redundant, linear and shifts in-variant . These properties allow the SWT to be realised using a recursive algorithm. Therefore, the SWT is a very useful algorithm for analysing a linear system. A brief description of the SWT is presented here. Figure 2 shows the computation of the SET of a signal $x(k)$, where $[W_j k]$ and $[V_j k]$ are called the detail and the approximation coefficients of the SWT. The filters H_j and G_j are the standard low-pass and high-pass wavelet filters, respectively. In the first step, the filters H_1 and G_1 are obtained by up-sampling the filters using the previous step (i.e., H_{j-1} and G_{j-1})

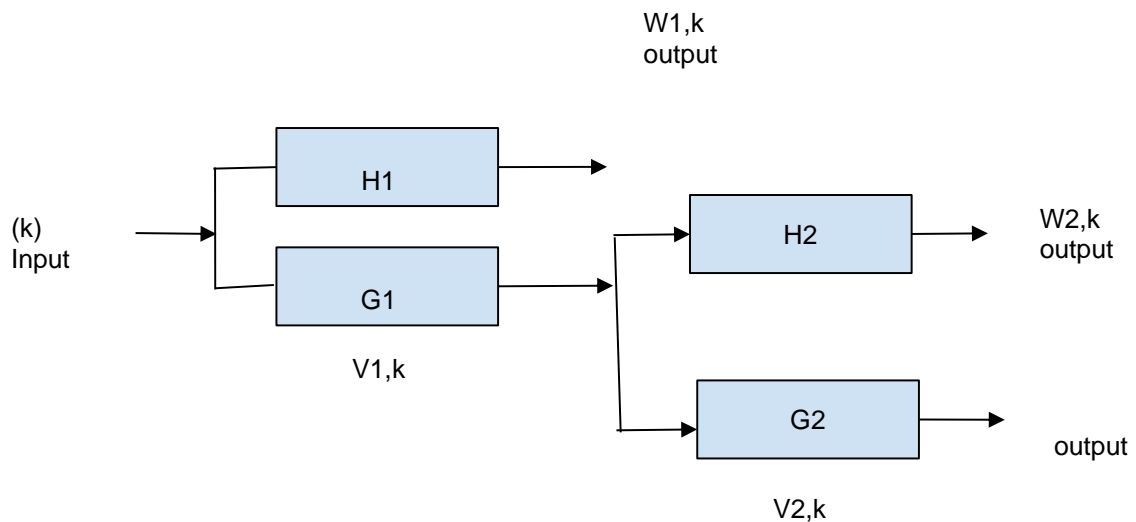


Fig 4.13 Illustration of Stationary wavelet transform

The detail coefficients W_j,k are equal to the output of the high-pass filters and similarly the approximation coefficients V_j,k are equal to the output of the low-pass filters.

According to $x(k)$ H_1 G_1 H_2 G_2 H_j G_j $W_{1,k}$ $V_{1,k}$ $W_{2,k}$ $V_{2,k}$ $W_{j,k}$ $V_{j,k}$ 4 the time frequency properties of the wavelet transform H_j and G_j are a bank of ideal narrowband filters.

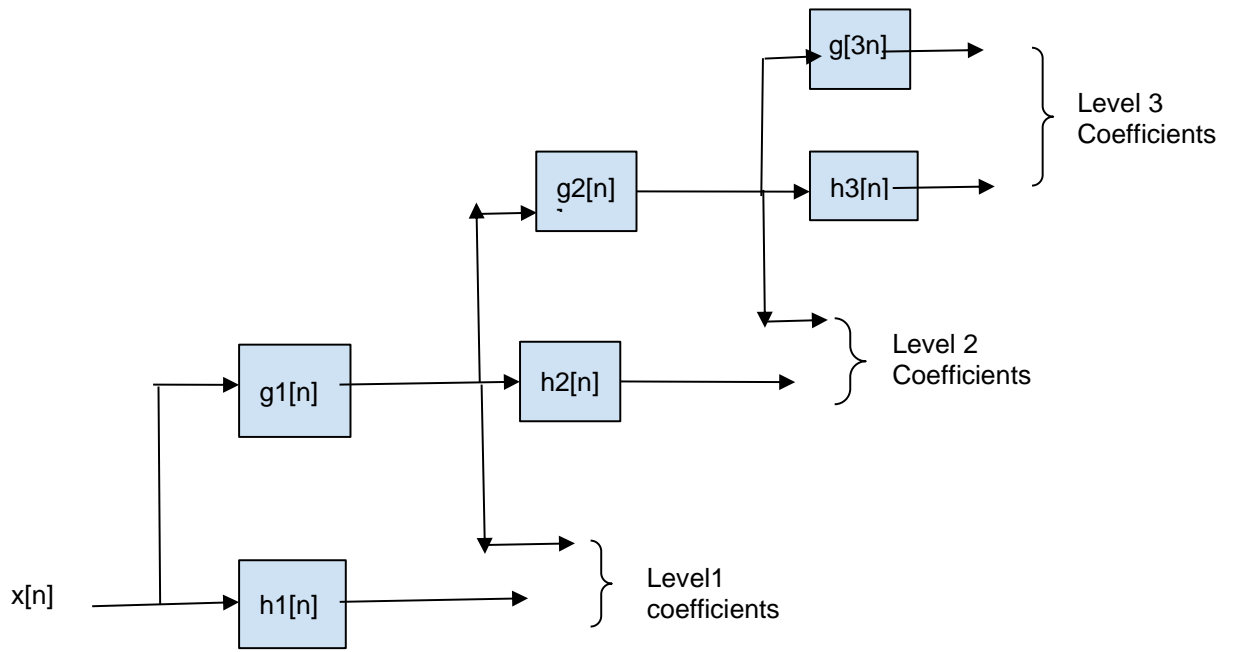


Fig 4.14 Block diagram of SWT

4.4.5 BIORTHOGONAL WAVELET TRANSFORM :

A Biorthogonal wavelet is a wavelet where the associated wavelet transform is invertible but not necessarily orthogonal. Designing biorthogonal wavelets allows more degrees of freedom than orthogonal wavelets. One additional degree of freedom is the possibility to construct symmetric wavelet functions.

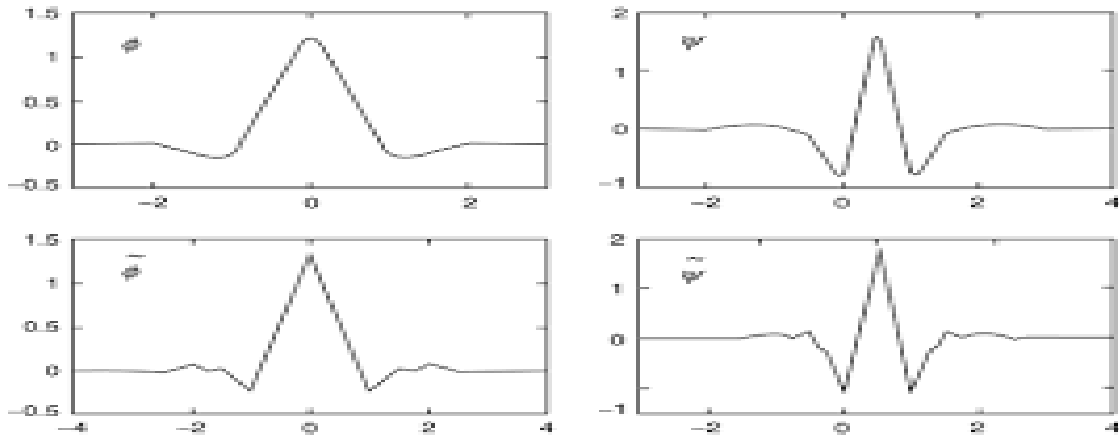


Fig 4.15 Biorthogonal Wavelets

It is well known that bases that span a space do not have to be orthogonal. To gain greater flexibility in the construction of wavelet bases, the orthogonality condition is relaxed allowing semi orthogonal, biorthogonal, or non-orthogonal wavelet bases. In the biorthogonal case, rather than having one scaling and wavelet function, there are two scaling functions that may generate different multiresolution analysis, and accordingly two different wavelet functions. The dual scaling and wavelet functions have the following properties:

1. They are zero outside of a segment.
2. The calculation algorithms are maintained, and thus very simple.
3. The associated filters are symmetrical

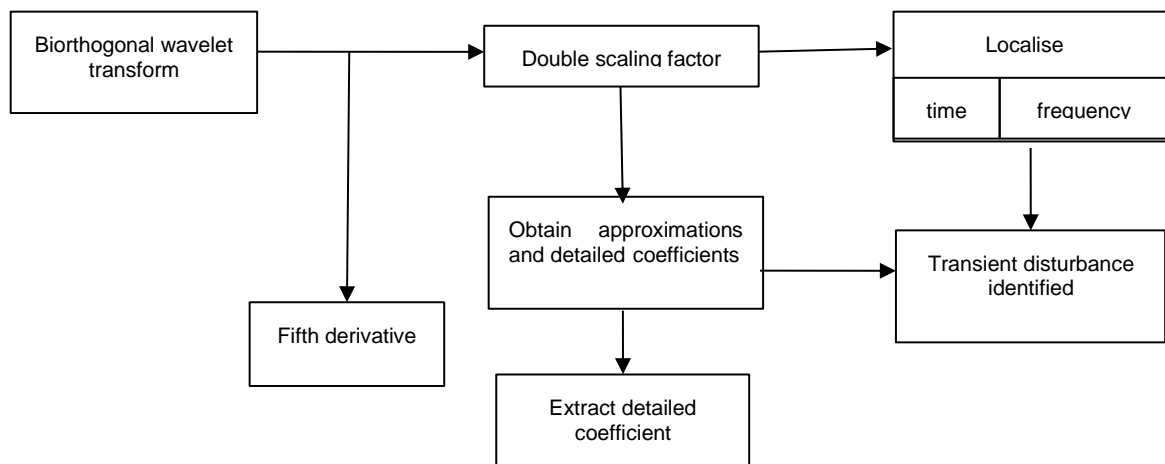


Fig 4.16 Procedural steps of Biorthogonal Wavelet Transform

4.5 GRAM-SCHMIDT TRANSFORM

The Gram-Schmidt process (or procedure) is a sequence of operations that allow us to transform a set of linearly independent vectors into a set of orthonormal vectors that span the same space spanned by the original set. The Gram-Schmidt process takes a finite, linearly independent set of vectors $S = \{v_1, \dots, v_k\}$ for $k \leq n$ and generates an orthogonal set $S' = \{u_1, \dots, u_k\}$ that spans the same k -dimensional subspace of \mathbf{R}^n as S .

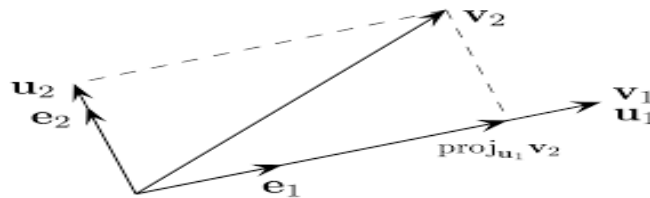


Fig 4.17 Gram-Schmidt Transform projection

The method is named after Jorgen Pedersen Gram and Erhard Schmidt, but Pierre-Simon Laplace had been familiar with it before Gram and Schmidt. In the theory of Lie group decompositions it is generalised by the Iwasawa decomposition.

The application of the Gram-Schmidt process to the column vectors of a full column rank matrix yields the QR decomposition (it is decomposed into an orthogonal and a triangular matrix).

4.5.1 PROJECTIONS

We define the projection operator by

$$\text{Proj}_u(v) = \frac{(u,v)}{(u,u)} u$$

where $\langle u,v \rangle$ denotes the inner product of the vectors u and v . This operator projects the vector v orthogonally onto the line spanned by vector u . If $u = \mathbf{0}$, we define $\text{proj}_{\mathbf{0}}(v) = \mathbf{0}$, i.e., the projection map $\text{proj}_{\mathbf{0}}$ is the zero map, sending every vector to the zero vector.

The Gram–Schmidt process then works as follows:

$$U_1 = v_1 \qquad e_1 = \frac{u_1}{\|u_1\|} \qquad \text{----- (1)}$$

$$U_2 = v_2 - \text{proj}_1(v_2) \qquad e_2 = \frac{u_2}{\|u_2\|} \qquad \text{----- (2)}$$

$$U_3 = v_3 - \text{proj}_1(v_3) - \text{proj}_2(v_3) \qquad e_3 = \frac{u_3}{\|u_3\|} \qquad \text{----- (3)}$$

$$U_4 = v_4 - \text{proj}_1(v_4) - \text{proj}_2(v_4) - \text{proj}_3(v_4) \qquad e_4 = \frac{u_4}{\|u_4\|} \qquad \text{----- (4)}$$

$$\begin{array}{ccc} : & & : \\ : & & : \end{array}$$

$$U_k = v_k - \sum_{j=1}^{k-1} \text{proj}_j(v_k) \qquad e_k = \frac{u_k}{\|u_k\|} \qquad \text{----- (5)}$$

The sequence $\mathbf{u}_1, \dots, \mathbf{u}_k$ is the required system of orthogonal vectors, and the normalised vectors $\mathbf{e}_1, \dots, \mathbf{e}_k$ form an orthonormal set. The calculation of the sequence $\mathbf{u}_1, \dots, \mathbf{u}_k$ is known as Gram–Schmidt orthogonalization, while the calculation of the sequence $\mathbf{e}_1, \dots, \mathbf{e}_k$ is known as Gram–Schmidt orthonormalization as the vectors are normalised.

To check that these formulas yield an orthogonal sequence, first compute $\langle u_1, u_2 \rangle$ by substituting the above formula for \mathbf{u}_2 : we get zero. Then use this to compute $\langle u_1, u_3 \rangle$ again by substituting the formula for \mathbf{u}_3 : we get zero. The general proof proceeds by mathematical induction.

The Gram–Schmidt process also applies to a linearly independent countably infinite sequence $\{\mathbf{v}_i\}_i$. The result is an orthogonal (or orthonormal) sequence $\{\mathbf{u}_i\}_i$ such that for the natural number n : the algebraic span of $\mathbf{v}_1, \dots, \mathbf{v}_n$ is the same as that of $\mathbf{u}_1, \dots, \mathbf{u}_n$.

The Gram–Schmidt process can be stabilised by a small modification; this version is sometimes referred to as **modified Gram-Schmidt** or MGS. This approach gives the same result as the original formula in exact arithmetic and introduces smaller errors in finite-precision arithmetic. Instead of computing the vector \mathbf{u}_k as

$$u_k^{(1)} = v_k - \text{proj}_{u_1}(v_k) - \text{proj}_{u_2}(v_k) - \dots - \text{proj}_{u_{k-1}}(v_k) \quad \text{-----}(6)$$

It can be computed as:

$$u_k^{(1)} = v_k - \text{proj}_{u_1}(v_k) \quad \text{-----}(7)$$

$$u_k^{(2)} = u_k^{(1)} - \text{proj}_{u_2}(u_k^{(1)}) \quad \text{-----}(8)$$

$$u_k^{(k-2)} = u_k^{(k-3)} - \text{proj}_{u_{k-2}}(u_k^{(k-3)}) \quad \text{-----}(9)$$

$$u_k^{(k-1)} = u_k^{(k-2)} - \text{proj}_{u_{k-1}}(u_k^{(k-2)}) \quad \text{-----}(10)$$

$$e_k = \frac{u_k^{(k-1)}}{\|u_k^{(k-1)}\|} \quad \text{-----}(11)$$

PROPERTIES OF GRAM-SCHMIDT TRANSFORM

The following properties are gram-schmidt transform

- It is continuous
- It is orientation preserving
- It commutes with orthogonal maps
- For the Gram–Schmidt process as described above (sometimes referred to as "classical Gram–Schmidt") this loss of orthogonality is particularly bad; therefore, it is said that the (classical) Gram–Schmidt process is numerically unstable.
- the error is increases much faster in classical Gram-Schmidt than in modified Gram-Schmidt.
- It is interesting to note that the following formulation of Gram-Schmidt algorithm (which is really just a better implementation of classical Gram-Schmidt) is referred to as the modified Gram-Schmidt by Schwarz and Rutishauser

CHAPTER -5

SIMULATION TOOL

CHAPTER 5

SIMULATION TOOL

5.1 INTRODUCTION

MATLAB is now a full-featured technical computing environment, but it started as a simple “Matrix Laboratory.” Three men, J. H. Wilkinson, George Forsythe, and John Todd played important roles in the origins of MATLAB. Our account begins more than 50 years ago. Wilkinson was a British mathematician who spent his entire career at the National Physical Laboratory (NPL) in Teddington, outside London. Working on a simplified version of a sophisticated design by Alan Turing, Wilkinson and colleagues at NPL built the Pilot Automatic Computing Engine (ACE), one of Britain’s first stored-program digital computers. The Pilot ACE ran its first program in May 1950. Wilkinson did matrix computations on the machine and went on to become the world’s leading authority on numerical linear algebra.

MATLAB is a scientific programming language and provides strong mathematical and numerical support for the implementation of advanced algorithms. It is for this reason that MATLAB is widely used by the image processing and computer vision community. One can interactively segment image data, compare image registration techniques, and batch-process large data sets. Visualisation functions and apps let you explore images, 3D volumes, and videos; adjust contrast; create histograms; and manipulate regions of interest (ROIs).

5.2 MATHWORKS

The MathWorks, Inc. MathWorks is an American privately held corporation that specialises in mathematical computing software. Its major products include MATLAB and Simulink, which support data analysis and simulation. MathWorks produces mathematical computing software for engineers, scientists, mathematicians, and

researchers. MATLAB, the language of technical computing, is a programming environment for algorithm development, data analysis, visualisation, and numeric computation.

5.3 MATLAB FUNCTIONS

5.3.1 READ,WRITE and SHOW IMAGE

`imread()` function is used for reading images. If we run this function with required data, image is converted to a two-dimensional matrix (grey image is two-dimensional, but colour image is three-dimensional) with rows and columns including grey value in the each cell.

`I=imread('path/filename. File extension');`

- `imread()` function only needs an image file. If the result of `imread()` function is equal to a variable, a matrix variable (I) is created. File name, extension, and directory path that contains the image must be written between two single quotes. If script and image file are in the same folder, path is not necessary.

`I=imread('onion.jpg')`

- `imwrite()` function is used to create an image. This function only requires a new image file name with an extension. If the new image is saved to a specific directory, the path of directory is necessary.

`I=imwrite(I,'onion.jpg')`

Example :

```
clc;
clear all;
close all;
I=imread('onion.jpg');
imshow(I);
imwrite(I,'onion.jpg');
```

5.3.2. IMAGE REVERSE

Image reverse technique, where all elements of the matrix are replaced to be the top row elements to the bottom row and the bottom row elements to the top row. In other words, the image rotates on the vertical axis.

5.3.2.1 Image mirroring

Mirroring technique is the rotating of reversed image on the horizontal axis. In matlab Image Processing Toolbox has `imrotate()` function for rotating image. This function needs three properties which are image matrix variable, rotating angle, and interpolation method.

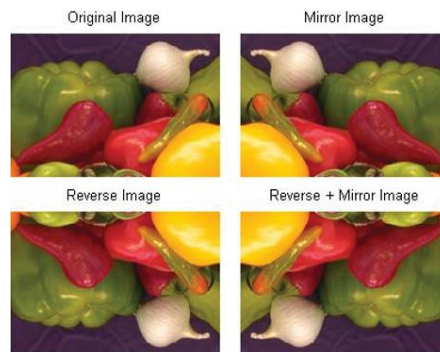


Fig 5.1 Image Reversal

5.3.2.2 Image shift:

An image can be wanted to shift up to certain pixel value on the horizontal and vertical axis. `imtranslate()` function is used to shift of an image.

Example:

```
I=imread('pout.tif');  
imshow(I);  
I_shift=imtranslate (I, [15,25]);  
figure;  
imshow(I_shift);
```

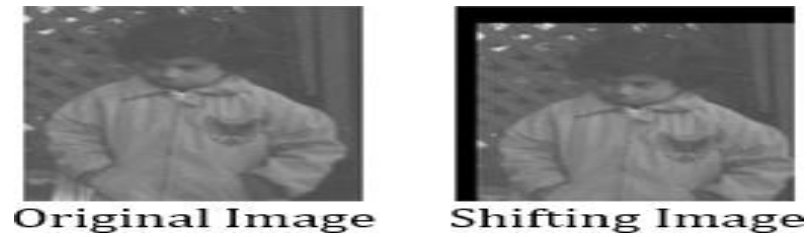


Fig 5.2 Shifted Image

5.3.3 GRAM-SCHMIDT TRANSFORM

Gram-Schmidt algorithm factorizes a matrix X into two matrix Q and R , where Q is an orthogonal or orthonormal matrix and R is a upper triangular matrix and $X=Q*R$. The difference between orthogonal and orthonormal matrix is that for an orthonormal matrix, each column is orthogonal to others and has unit length.

5.3.3.1 Gram-Schmidt orthogonalization

Gram-Schmidt orthogonalization, also called the Gram-Schmidt process, is a procedure which takes an orthogonal set of linearly independent functions and constructs an orthogonal basis over an arbitrary interval with respect to an arbitrary weighting function.

$$gsog(x)$$

5.3.3.2 Gram-Schmidt orthonormalization

The Gram–Schmidt process is a method for orthonormalizing a set of vectors in an inner product space, most commonly the Euclidean space R_n equipped with the standard inner product.

$$gson(x)$$

5.4 MATLAB TOOLBOX

The toolbox is a collection of functions built on the MATLAB® technical computing environment.

The toolbox can be used to:

- Connect to CAN Devices.
- Use Supported CAN Devices and Drivers.
- Communicating Between MATLAB and CAN Bus.
- Simulate CAN Communication.
- Visualise CAN Communication
- Installation of toolbox

In MATLAB, go to the home tab. 2. Select Add-Ons > Manage Add-Ons. MATLAB displays a list of MathWorks products, toolboxes, and add-ons installed on your machine.

The Image Processing Toolbox provides a set of functions and applications for image processing, analysis, and visualisation. There are many functions available for image analysis, image segmentation, image enhancement, noise reduction, geometric transformations, and image registration.

Wavelet Toolbox provides apps and functions for analysing and synthesising signals and images. You can detect events like anomalies, change points, and transients, and denoise and compress data.

Wavelet Toolbox can interactively denoise signals, perform multiresolution and wavelet analysis, and generate MATLAB code. The toolbox includes algorithms for continuous and discrete wavelet analysis, wavelet packet analysis, multiresolution analysis, wavelet scattering, and other multiscale analysis.

5.4.1 WAVELET TOOLBOX FUNCTIONS:

- **dwt2**

`dwt2` computes the single-level 2-D wavelet decomposition. Compare `dwt2` with `wavedec2` which may be more useful for your application. The decomposition is done with respect to either a particular wavelet (see `wfilters` for more information) or wavelet decomposition filters.

- `[cA,cH,cV,cD] = dwt2(X,wname)` computes the single-level 2-D discrete wavelet transform (DWT) of the input data `X` using the `wname` wavelet. `dwt2` returns the approximation coefficient matrix `cA` and detail coefficient matrices `cH`, `cV`, and `cD` (horizontal, vertical, and diagonal, respectively).
- `[cA,cH,cV,cD] = dwt2(X,LoD,HiD)` computes the single-level 2-D DWT using the wavelet decomposition lowpass filter `LoD` and highpass filter `HiD`. Decomposition filters must have the same length and an even number of samples.
- `[cA,cH,cV,cD] = dwt2(_, 'mode', extmode)` computes the single-level 2-D DWT with the extension mode `extmode`. Include this argument after all other arguments.

- **Haar2**

A Haar wavelet is a sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal basis.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions (discrete signals), such as monitoring of tool failure in machines.

CHAPTER -6
PROJECT DESCRIPTION

CHAPTER 6

PROJECT DESCRIPTION

In this project first we have taken two images namely SAR and optical images. Our main idea is to fuse SAR and optical image to test the effectiveness of image fusion. Now we take these two original images and apply discrete wavelet transforms and then apply Gram-Schmidt transforms and feature-level fusion to get the fused image. By using the maximum selection fusion rule we fuse the two images and then we apply an inverse wavelet and inverse Gram-Schmidt transform to get the fused image. For calculating the performance we compare the parameters such as information entropy, average gradient, and cross entropy of the base paper. We compare these parameters with the parameters of the wavelet and Gram-Schmidt transform fused image.

6.1 WAVELET DECOMPOSITION

Apply wavelet transform to the image. Here a low pass and high pass filter is applied to the columns and then it is applied to the rows, we'll get four bands. Approximation image(LL), vertical details(LH), horizontal details(HL) and diagonal details(HH)of the two images. Above block diagram shows first level and second level decomposition.

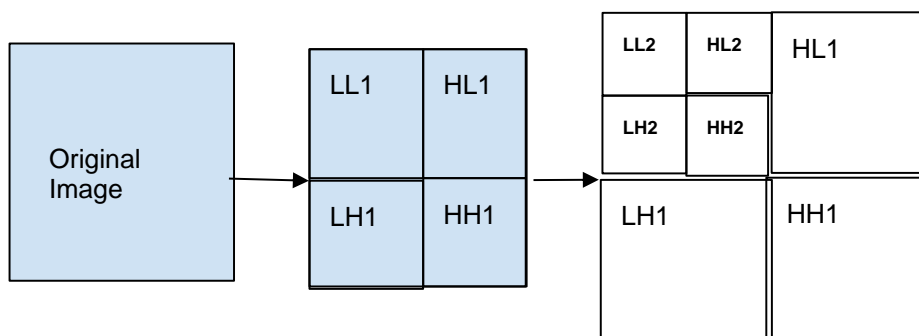


Fig 6.1 Wavelet decomposition of the image

6.2 WAVELET TRANSFORM

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines.

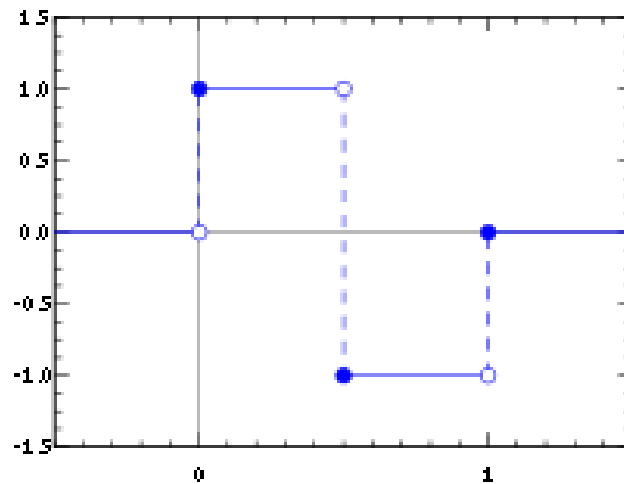


Fig 6.2 Haar wavelet transform

The Haar wavelet mother wavelet function is defined as

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Its scaling function is described as:

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

6.3 GRAM-SCHMIDT TRANSFORM

In the process, a lower spatial resolution pan image is simulated and a Gram-Schmidt transformation is performed on the simulated lower spatial resolution pan image and plurality of lower spatial resolution spectral band images, wherein the simulated lower spatial resolution Pan image is employed as the first band in the Gram-Schmidt transformation.

This method has two main advantages over previous pan-sharpening techniques. First, there is no limit to the number of bands that can be processed at one time using this

technique. Secondly, the spectral characteristics of the lower spatial resolution multispectral data are preserved in the higher spatial resolution, pan-sharpened multispectral data.

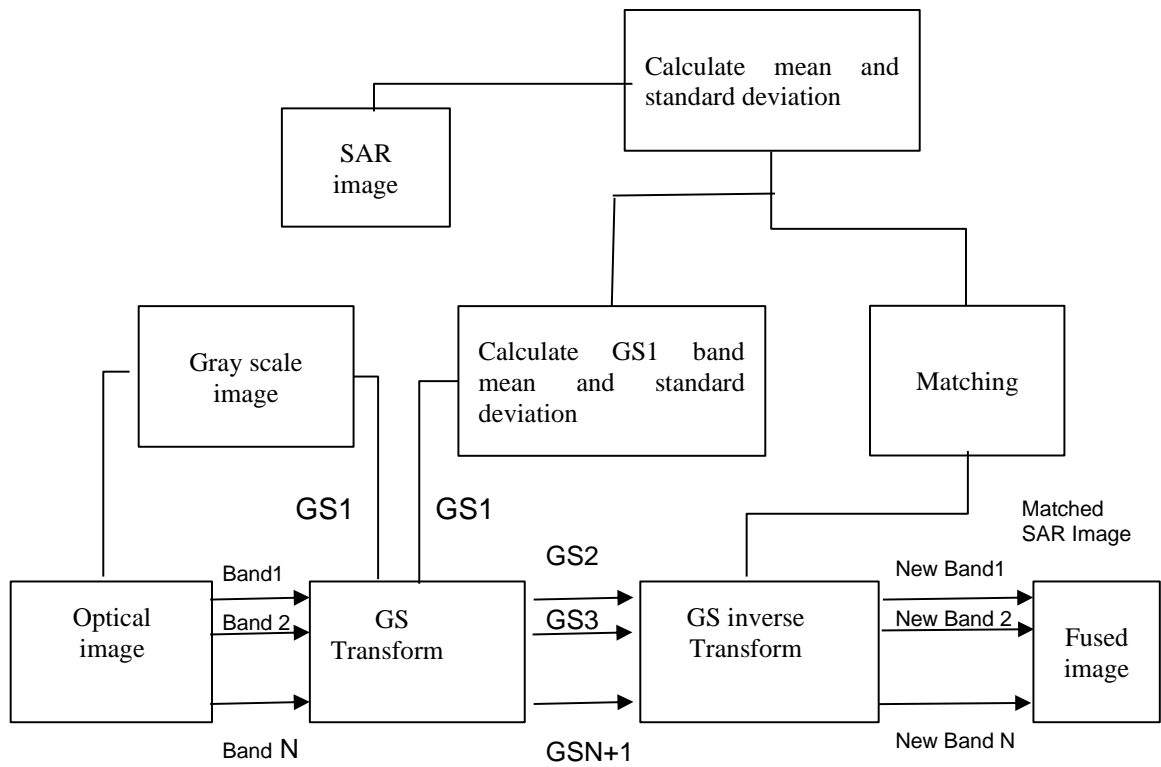


Fig 6.3 Gram-Schmidt Flowchart transform

6.4 FUSION TECHNIQUE:

Maximum Selection Scheme:

This scheme just picks coefficients in each sub-band with smallest magnitude. A selection process is performed here wherein, for every corresponding pixel in input images the pixel with maximum intensity is selected and is put in as the resultant pixel of the fused image.

$$K(i, j) = \max[w(I_1(x, y)), w(I_2(x, y))]$$

where $I_1(x, y)$ and $I_2(x, y)$ are the input images.

6.5 FUSION FLOWCHART:

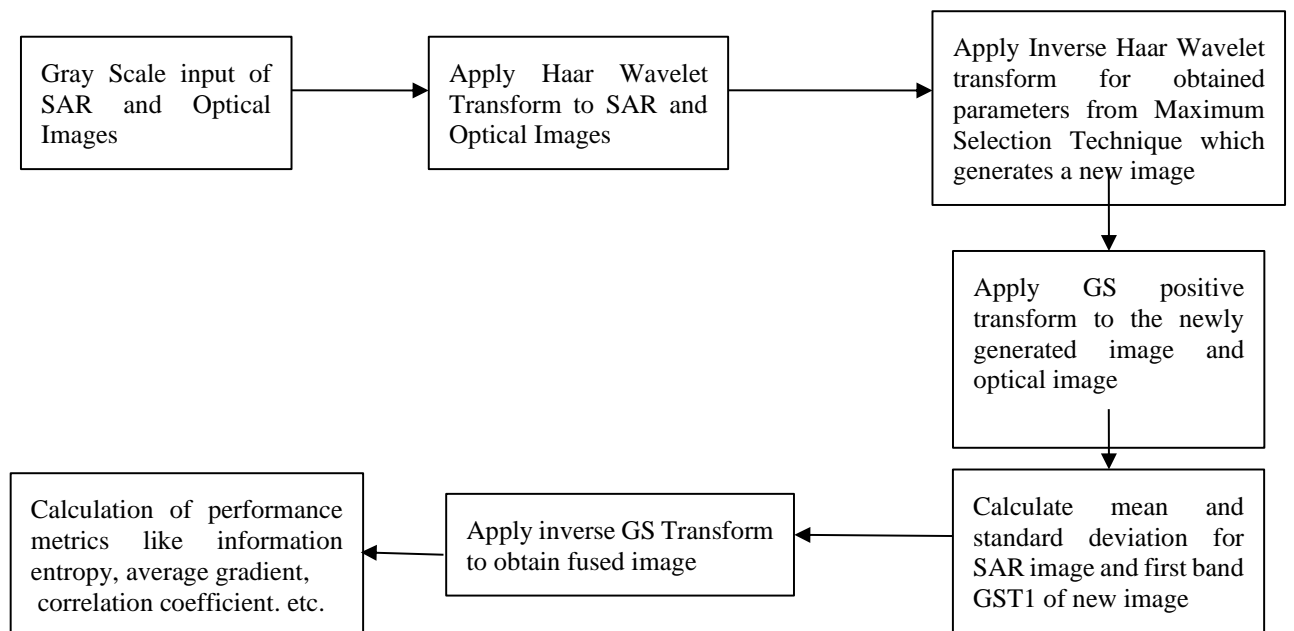


Fig 6.4 Image Fusion method combining Gram-Schmidt and Haar Wavelet Transform

6.6 ALGORITHM

- STEP-1: We consider the input images SAR and optical which are in grayscale.
- STEP-2: Then we apply Haar wavelet transform to both the grayscale SAR and optical images.
- STEP-3: Now apply the inverse haar wavelet transform for the obtained from selection technique which generates a new image.
- STEP- 4: Consider the newly obtained image and apply GS positive transform to that image.
- STEP- 5: Next we tend to calculate the mean and standard deviation for SAR image and first band of GST1 of the obtained new image.
- STEP- 6: And now apply the inverse GS transform to obtain the fused image.
- STEP-7 : Lastly calculation of performance metrics like information entropy, average gradient and correlation coefficient etc. is done to compare with Haar wavelet transform.

6.7 PERFORMANCE ANALYSIS:

Information entropy:

Information entropy is an important indicator to measure the richness of image information. The information entropy of wavelet transform is the lowest because of the severe spectral distortion, which makes it difficult to show details but when it comes to the fusion of gram-schmidt transform and wavelet transform the entropy is relatively high

$$h = p_1 \log_s \left(\frac{1}{p_1} \right) + p_2 \log_s \left(\frac{1}{p_2} \right) + \dots + p_k \log_s \left(\frac{1}{p_k} \right)$$

Cross entropy:

The cross entropy reflects the information difference between the grey value distribution of the two images. The cross entropy in this paper is the total cross entropy

of the fusion image and the two original images, that is, the squared average of the two cross entropies. The smaller the cross entropy, the smaller the difference between the fused image and the two original images. It can be seen that the cross entropy of our method is the smallest. This also reflects that the fusion image has the richest information compared to other methods.

$$E = \sum_{i=0}^{l-1} P(i) \log_2 P(i)$$

Average gradient :

The average gradient (AG) can reflect the performance of the fusion image, the improvement of image quality and the characteristics of small details, contrast and texture changes in the image. The imaging characteristics of the SAR image itself give it has rich information on the change of the surface height, which is reflected on the higher average gradient. The AG of the optical image reflects the gradient change of the spectrum, which is not as dramatic as the SAR image.

$$A_G = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N \sqrt{\frac{f_x^2(i,j) + f_y^2(i,j)}{2}}$$

Correlation coefficient:

The correlation coefficient (CC) reflects the correlation between the two images. The closer the correlation is to 1, the closer the two images are. The CC in this paper is the average of the square of the fusion image with the optical image and SAR image respectively. This index reflects that the fusion image of our method has a strong correlation with the two source images. It reflects that our method can better preserve the spectrum than other methods.

$$C_C = \frac{\sum_{i=1}^M \sum_{j=1}^N [p_m(a,b) - \underline{p}_m(a,b)] \times [p_n(a,b) - \underline{p}_n(a,b)]}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n [p_m(a,b) - \overline{p}_m(a,b)]^2 \times [p_n(a,b) - \overline{p}_n(a,b)]^2}}$$

CHAPTER -7

RESULTS

CHAPTER-7

RESULTS

7.1 Parameter Analysis and Output Fused Images

Transform Parameters	Gram-schmidt with wavelet transform	Haar wavelet transform
Entropy	7.5467	7.5109
Cross entropy	0.0081	0.0768
Correlation coefficient	0.4758	0.3842
Average gradient	65.2644	62.6073

Table 7.1 Parameter Analysis of set 1 Images

SET1



(a) Optical Image



(b) SAR Image



(c) GS with Haar Transform Fused output Image



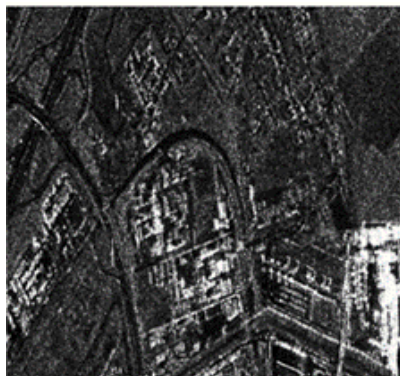
(d) Haar Transform Fused Output Image

Fig 7.1 Original and Fused images of set 1 images

Transform Parameters	Gram-schmidt with wavelet transform	Haar wavelet transform
Entropy	7.4120	7.3964
Cross entropy	0.0339	0.1397
Correlation coefficient	0.6426	0.5472
Average gradient	71.6074	60.8483

Table 7.2 Parameter Analysis of set 2 Images

SET2



(a) SAR Image



(b) Optical Image



(c) GS with Haar Transform Fused Output image



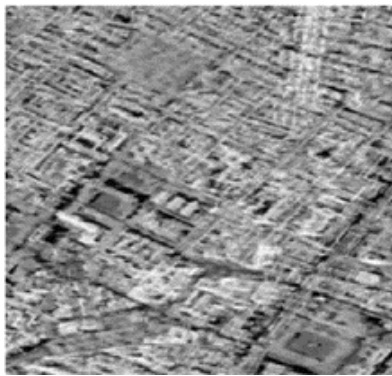
(d) Haar Transform Fused Output Image

Fig 7.2 Original and Fused images of set 2 images

Transform Parameters	Gram-schmidt with wavelet transform	Haar wavelet transform
Entropy	7.5403	7.5529
Cross entropy	0.0253	0.1383
Correlation coefficient	0.4653	0.3842
Average gradient	63.4742	48.0443

Table 7.3 Parameter Analysis of set 3 Images

SET3



(a) SAR image



(b) Optical image



(c) GS with Haar Transform Fused Output Image



(d) Haar Transform Fused Image

Fig 7.3 Original and Fused Images of set 3 images

CONCLUSION

The aim of this work has been to compare Haar wavelet transform with a combination of Gram-Schmidt and wavelet transform fusion techniques for image fusion. For an effective fusion of images, a technique should aim to retain important features from all input images. These features often appear at different positions and scales. Multiresolution analysis tools such as the wavelet are therefore ideally suited to image fusion. The proposed methods which were the main tool for image fusion, that yields good sharpness and clarity along with increase in information, are well suited for fusion of remote-sensing images. Fusion is performed for gray scale images and performance characteristics between the fused images of the two methods are compared based on different image parameter metrics. The fused output images have the characteristics of both the input images in which both details are visible properly. From the parameters, it was observed that using Gram Schmidt with Haar wavelet transform technique is more effective than Haar wavelet transform, which is evident through the values of the parameters obtained for cross entropy, average gradient, correlation coefficient and Information entropy.

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